Foundation Maths for First Year Physics — M. Coppins, 14.10.04

Study Guide and Problem Sheet/Classwork
Lecture 4: Trigonometry and Geometry

Learning Outcomes

Jargon

Identity, cartesian coordinates, arc, sector, quadrant, periodic function.

Concepts

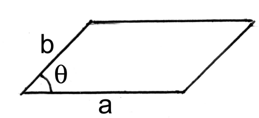
Identity linking $\sin^2\theta$ and $\cos^2\theta$; finding the distance of a given point from the origin; finding the distance between two points and the gradient of the line joining them; finding the equation of a straight line given two points on it; the equation of a circle; converting from degrees to radians and vice versa; finding the length of an arc; finding the area of a sector; finding the area of a triangle; the form of the sin, cos and tan functions for all angles; how sin/cos of $\pi/2-\theta$ are related to sin/cos of θ; small angle approximations for sin and cos.

Problems

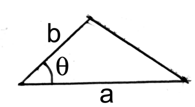
1. By considering a right angled triangle formed by splitting an equilateral triangle of side 2 in half, find exact values for sin, cos and tan of 30^o and 60^o.

2. (a) Convert the following angles into radians: (i) 45^o, (ii) -10^o, (iii) 720^o. (b) Convert the following angles (in radians) into degrees: (i) 0.1, (ii) 6π, (iii) $-7\pi/2$.

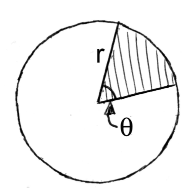
3. (a) A parallelogram has adjacent sides a and b and the included angle is θ. Show that the area is $ab\sin\theta$.



   (b) A triangle has adjacent sides a and b and the included angle is θ. Show that the area is $\frac{1}{2}ab\sin\theta$.



   (c) A sector of a circle (shown shaded in the figure) of radius r subtends an angle θ at the centre. Show that if θ is in radians the area of the sector is $\frac{1}{2}r^2\theta$.

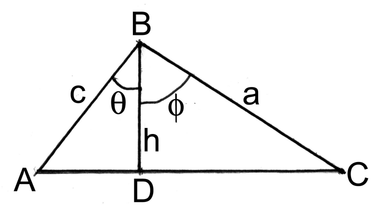


4. The reciprocal trigonometric functions cosecant, secant and cotangent are defined as follows: $\text{cosec }\theta = 1/\sin\theta$, $\sec\theta = 1/\cos\theta$, $\cot\theta = 1/\tan\theta$.

   (a) Write down: cosec 45^o, sec 30^o, cot 60^o.

   (b) Prove the following identity: $\tan^2\theta + 1 = \sec^2\theta$.

5. (a) With reference to the figure show that: area of triangle ABD is $\frac{1}{2}ac\sin\theta\cos\phi$, area of triangle DBC is $\frac{1}{2}ac\cos\theta\sin\phi$, area of triangle ABC is $\frac{1}{2}ac\sin(\theta+\phi)$, and, hence, show that: $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$.



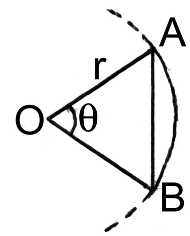
(b) By replacing θ by $\pi/2-\theta$ and ϕ by $-\phi$, show that: $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$.

(c) Hence, show that: $\cos\theta = 1 - 2\sin^2(\theta/2)$.

6. (a) With reference to the figure show that:

$$\frac{\text{area of triangle OAB}}{\text{area of sector OAB}} = \frac{\sin\theta}{\theta}$$

where θ is in radians.



(b) By considering what happens to the ratio of areas as θ is made very small, show that for small angles $\sin\theta \simeq \theta$.

(c) Using the identity $\cos\theta = 1 - 2\sin^2(\theta/2)$. (Q 5, above) and the small angle approximation for $\sin\theta$, show that for small angles $\cos\theta \simeq 1 - \theta^2/2$.

(d) Is the following statement true or false: these small angle approximations give values of sin and cos accurate to at least 5 decimal places for an angle of 1^o.

7. Point A has coordinates (2,2). Point B has coordinates (3,4). Find:

(a) the distance from the origin to A,

(b) the distance from the origin to B,

(c) the distance between A and B,

(d) the gradient of the straight line which passes through A and B,

(e) tan of the angle between the x axis and the straight line through A and B,

(f) the equation of the straight line through A and B.

8. (a) Write down an expression for the distance of an arbitrary point, coordinates (x, y) from a point with coordinates (a, b). Hence show that the equation of circle of radius r, centred at (a, b), has the form $x^2 + y^2 + \alpha x + \beta y = \gamma$ and find expressions for α, β and γ in terms of a, b and r.

(b) Find the radius and centre of the circles specified by the following equations:
(i) $x^2 + y^2 - 2x = 3$
(ii) $x^2 + y^2 + 4x - 8y = -11$
(iii) $x^2 + y^2 + 2x + 4y = -9$