Foundation Maths for
First Year Physics

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Study Guide and Problem Sheet/Classwork
Lecture 3: Functions I

Learning Outcomes

Jargon

Function, independent variable, dependent variable, argument of function, x or y axis intercept, linear function, quadratic function, gradient, even and odd functions.

Notation

$f(x)$, Δx

Concepts

Familiarity with the equations and graphs of linear and quadratic functions; connection between the sign of the gradient and the direction of the slope; effect of the following transformations on the curve $y = f(x)$: $f(x + \alpha)$, $f(x) + \alpha$, $\alpha f(x)$, $f(\alpha x)$; effect of plotting function of form $f(x) = ax^b$ on log-log scales.

Problems

1. Given $f(x) = 2x + 4$ write down:

   (a) $f(0)$ (b) $f(2x)$ (c) $f(-2x)$ (d) $f(1/x)$ (e) $f(3a - 2)$

2. An *even* function has the property that $f(x) = f(-x)$. An *odd* function has the property that $f(x) = -f(-x)$. Decide if the following functions are even, odd, or neither:

   (a) $f(x) = x$ (b) $f(x) = x^2$ (c) $f(x) = x^2 + 2$ (d) $f(x) = x^2 + x$
   (e) $f(x) = (x + 2)^2$

3. Find the equation of the straight line (in the form $y = ax + b$) which crosses the y axis at $y = 2$ and has a gradient of:

   (a) 1 (b) 4 (c) -2 (d) 0 (e) ∞

4. (a) Sketch the following functions for x in the range $-2 \leq x \leq 2$ (all three on the same set of axes): $y = x$, $y = x^2$, $y = x^3$.

   (b) Sketch the following functions for x in the range $-2 \leq x \leq 2$ (all three on the same set of axes): $y = x^{-1}$, $y = x^{-2}$, $y = x^{-3}$.

5. In the lecture we found that the graph of $y = \gamma\{(x+\alpha)^2+\beta\}$ has the following properties: (i) y has an extreme value of $\gamma\beta$, (ii) this extreme value is a minimum if $\gamma > 0$, a maximum if $\gamma < 0$, (iii) the curve crosses the x axis if $\beta < 0$.

   What can you deduce about the form of the graph of the quadratic function $y = ax^2 + bx + c$? (i.e., rewrite the above properties in terms of a, b and c instead of α, β and γ).

6. (a) Given the function $f(x) = 2x + 4$, sketch graphs of: $y = f(x)$, $y = f(2x)$, $y = f(-2x)$.

   (b) How would you describe the way in which the curve $y = f(x)$ is transformed into $y = f(\alpha x)$?

   (c) In the lecture transformations of curves were illustrated using the function $f(x) = x^2$. Why do you think a different function has been chosen here to illustrate the transformation $f(x) \to f(\alpha x)$? [Hint: your answer should include the word *even*.]

7. (a) Given $y = ax^b$, show that a graph of $Y = \log_{10} y$ against $X = \log_{10} x$ is a straight line, and determine the gradient of the line and its Y axis intercept.

   (b) For some function $y = f(x)$, a graph of $Y = \log_{10} y$ against $X = \log_{10} x$ is a straight line of gradient -0.5 which intercepts the Y axis at $Y = -1$. What is $f(x)$?

   (c) For a certain type of function $y = g(x)$ the graph of $Y = \log_{10} y$ against x (*not* X) is a straight line. Assuming that this straight line has a gradient of α and intercepts the Y axis at $Y = \beta$, deduce the form of $g(x)$.

8. Decide if the following statements are true or false:

   (a) When plotting the graph of a function the independent variable is usually plotted on the horizontal axis.

   (b) If $f(x)$ is zero at $x = 5$ then $f(x - 3)$ must be zero at $x = 2$.

   (c) $f(1/x)$ is the same thing as $1/f(x)$.

   (d) The function $f(x) = -2x^3$ gives positive values of $y = f(x)$ for all negative values of x.

   (e) The lines $y = 2x + 1$ and $y = \frac{1}{2}(7 + 4x)$ are parallel.