Foundation Maths for First Year Physics

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Study Guide and Problem Sheet/Classwork
Lecture 2: Basic Algebra

Learning Outcomes

Jargon

Like terms, coefficient, expansion, polynomial, order of a polynomial, binomial expansion, sequence, recurrence relation, series.

Notation

$$\sum_{j=1}^{n}$$

Concepts

Removing brackets from algebraic expressions; familiarity with expansions of $(a+b)^2$, $(a-b)^2$ and $(a+b)(a-b)$; using Pascal's triangle to find coefficients of a binomial expansion; rationalizing the denominator of a fraction involving surds; obtaining partial fractions for some simple algebraic fractions.

Problems

1. Find the coefficient of x^2 in the following expressions:

   (a) $(3x+2)^2$ (b) $(x+2)(x+1)^2$ (c) $x(1+x)^{10}$ (d) $x^6\left(1-\frac{1}{x}\right)^4$

   (e) $(ax+b)^3$

2. Factorize the following expressions:

   (a) $2x^2+4x+2$ (b) $a^2x^3+2abx^2+b^2x$ (c) $16x^2-9$

   (d) $x^4-2a^2x^2+a^4$ (e) $x^2-(a+b)x+ab$

3. Write the following fractions with rational denominators:

   (a) $\frac{1}{\sqrt{5}-\sqrt{2}}$ (b) $\frac{3}{2+\sqrt{3}}$ (c) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ (d) $\frac{3+5\sqrt{2}}{4-2\sqrt{2}}$

   (e) $\frac{\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}$

4. A general second order polynomial ax^2+bx+c can be written in the form $\gamma\{(x+\alpha)^2+\beta\}$. Find expressions for α, β and γ in terms of a, b and c.

5. Expand $\left(2+\frac{p}{2}\right)^4$.

6. The expression $\frac{3x+1}{(x+2)(x-3)}$ can be written as $\frac{A}{(x+2)} + \frac{B}{(x-3)}$. This way of splitting up an expression with a denominator consisting of the product of two factors is called decomposing into *partial fractions*.

   (a) Show that for any x: $A(x-3) + B(x+2) = 3x+1$

   (b) Substitute $x = -2$ to find A.

   (c) Substitute an appropriate value of x to find B.

   (d) Express $\frac{x+4}{(x+3)(x-5)}$ in partial fractions.

7. Write out the first four terms of the following series:

   (a) $\sum_{j=1}^{\infty} 2^j$ (b) $\sum_{j=0}^{\infty} \left(-\frac{1}{2}\right)^j$ (c) $\sum_{j=2}^{\infty} jx^{j-2}$ (d) $\sum_{r=1}^{\infty} (-1)^{r+1} rx^r$

   (e) $1 + \sum_{j=1}^{\infty} (-1)^j (j+1) x^j$

8. A *geometric progression* (GP) is a sequence in which each term is a constant multiple of the preceeding term. This constant multiple is called the *common ratio.* A GP has n terms, the first term is a, and the common ratio is r. The sum of the n terms is denoted S_n.

   (a) Write down the first three terms and the last term.

   (b) Show that $S_n - rS_n = a - ar^n$, and, hence, that $S_n = a\frac{(1-r^n)}{1-r}$.

   (c) Show that if $|r| < 1$ the sum of the GP with an infinite number of terms is $S_\infty = \frac{a}{1-r}$.

   (d) Hence show that $(1-x)^{-1}$ (for $|x| < 1$) can be written as the infinite series $1 + x + x^2 + x^3 + x^4 + ...$