Foundation Maths for First Year Physics — M. Coppins
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Study Guide and Problem Sheet/Classwork
Lecture 1: Numbers

Learning Outcomes

Jargon

Rational number, irrational number, prime number, ratio, factorize, factorial, numerator, denominator, index, logarithm to base a.

Notation

$n!$, $\log_a c$

Concepts

Manipulating fractions; combining indices; the connection between indices and logarithms; manipulating logarithms.

Problems

1. Decide if the following numbers are rational or irrational:

   (a) $\dfrac{37}{103}$ (b) $\dfrac{\pi}{103}$ (c) -5.137 (d) $\sqrt{2}+\dfrac{4}{7}$ (e) $\sqrt{32}$

2. Find the prime factors of the following numbers:

   (a) 42 (b) 43 (c) 44 (d) 625 (e) 6!

3. A cake is divided into two pieces in the ratio 3:2. What percentage of the cake is the larger piece?

4. Simplify the following fractions (i.e., write them in terms of a common, rational denominator):

   (a) $\dfrac{1}{7}+\dfrac{1}{5}$ (b) $\dfrac{1006}{503}+\dfrac{14}{63}$ (c) $\dfrac{1}{\left(\frac{7}{6}-1\right)}-\dfrac{1}{\left(\frac{3}{2}-\frac{5}{6}\right)}$ (d) $\dfrac{5}{\sqrt{3}}-\dfrac{8}{\sqrt{2}}$

   (e) $\dfrac{a}{b^2}-\dfrac{ac}{2b}$

5. Simplify the following:

   (a) $\left(\dfrac{1}{(2^4)^{-1/2}}\right)^3$ (b) $\left(\dfrac{1}{7}\right)^0+\left(\dfrac{1}{7}\right)^{-0.5}\times\left(\dfrac{1}{7}\right)^{-1.5}$ (c) $\dfrac{(x^3)^{1/2}}{(x^2)^{1/3}}$

   (d) $(3^x)^2\times\left(\dfrac{1}{3}\right)^{-4x}$ (e) $\left(\dfrac{a^5}{b^{-2}}\right)^{1/4}\times\dfrac{b^{3/2}}{a^2}$

6. By writing $b = a^x$ and $c = a^y$ prove the following laws of logarithms:
   (a) $\log_a(bc) = \log_a b + \log_a c$
   (b) $\log_a(b/c) = \log_a b - \log_a c$
   (c) $\log_a(b^n) = n \log_a b$

7. Decide if the following statements are true or false:
   (a) $\log_2 16 = 4$
   (b) $\log_x 5 = y$ implies that $y^x = 5$
   (c) $3 \log_a 4 + \log_a(4^{-1}) - \log_a 2 = \log_a 32$
   (d) the logarithm of 1 to any base is zero
   (e) $\log_a x$ is negative if $0 < x < 1$

8. Show that $\log_a c = \dfrac{\log_{10} c}{\log_{10} a}$. [Hint: take the log to base 10 of $c = a^b$.]