Problems for Lecture 9: Determinants

1. Evaluate (a) $\begin{vmatrix} 4 & 1 & 2 \\ 7 & 2 & 0 \\ -2 & 3 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 3 & 2 & 4 \\ 5 & 4 & 8 \\ 8 & 2 & 9 \end{vmatrix}$ (c) $\begin{vmatrix} 2 & 15 & -37 & 8 & 11 \\ 0 & 1 & 6 & 23 & -32 \\ 0 & 0 & 4 & 12 & -29 \\ 0 & 0 & 0 & 10 & 20 \\ 0 & 0 & 0 & 0 & 3 \end{vmatrix}$

2. Determine which of the following determinants are zero. For each determinant that is zero, try to identify what characteristic of the determinant ensures that it is so.

    (a) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 6 & 12 \\ -5 & 10 & -15 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & -1 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 7 & 3 & 2 \\ 6 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 0 & 7 & 0 \\ 3 & -5 & 6 \\ 2 & 3 & -4 \end{vmatrix}$

3. Show that the following determinants are zero:

    (a) $\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$

    Both determinants can be shown to be zero by multiplying them out, but the point of this question is rather to obtain the results *by using the general properties of determinants.*