Problems for Lecture 7: Linear Equations & Determinants

1. Find the equation of the plane that is normal to the vector $\mathbf{n} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and that passes through the point $\mathbf{a} = (1, -2, 3)$.

2. (a) If the points A = (1, 1, *a*), B = (2, *b*, 7) and C = (*c*, 5, –5) all lie on the plane specified by the equation $4x - 3y + 2z = 7$, find *a*, *b* and *c*.

   (b) Find a unit vector normal to the plane defined in part (a).

   (c) Verify that the vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$ from part (a) are perpendicular to the normal vector found in part (b).

3. Evaluate the determinants (a) $\begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix}$ and (b) $\begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$. How are the determinates related?

   Now evaluate each of the following 2×2 determinants, and identify how each is "developed" from the determinants in (a) and (b) above:

   (c) $\begin{vmatrix} 1 & 5 \\ 4 & 2 \end{vmatrix}$ (d) $\begin{vmatrix} 8 & 2 \\ 2 & 5 \end{vmatrix}$ (e) $\begin{vmatrix} 4 & 2 \\ 5 & 7 \end{vmatrix}$

4. Evaluate the determinant $\begin{vmatrix} a & b \\ ca & cb \end{vmatrix}$ where $c \neq 0$. What can you conclude in general?

5. For each of the following four pairs of equations, identify those that have a unique solution. Use Cramer's rule to solve those that do. For the equations that do not have a unique solution, identify whether they have no solutions or infinitely many solutions.

   (a) $\begin{aligned} 3x + 5y &= 14 \\ 2x + 4y &= 10 \end{aligned}$

   (b) $\begin{aligned} 3x - 5y &= 8 \\ 7x + 2y &= 12 \end{aligned}$

   (c) $\begin{aligned} 6x + 3y &= 9 \\ 4x + 2y &= 6 \end{aligned}$

   (d) $\begin{aligned} 1.4x - 1.2y &= 6.4 \\ -2.1x + 1.8y &= -4.7 \end{aligned}$