Problems for Lecture 6: Geometry

In questions 1-3, we consider the two-dimensional space $\mathbb{R}^2$.

1. Write down the vector equation of a straight line through $\mathbf{r}_1 = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{r}_2 = 8\mathbf{i} - 5\mathbf{j}$.

   Show that the equation can be written in the form $\dfrac{x-3}{5} = -\dfrac{y-4}{9}$.

2. Write down the vector equation of a straight line of gradient 3 with an intercept on the *y*-axis at $y = -2$. Obtain the Cartesian (*x*-*y*) form as well.

3. For the lines of questions 1 and 2, find

   (a) the direction ratios,

   (b) the direction cosines,

   (c) the unit normal vectors,

   (d) the angle between the two lines,

   (e) the angle between the two normals,

   (f) the perpendicular distances from the origin.

In questions 4-5 we consider the three-dimensional space $\mathbb{R}^3$.

4. Write down the vector equation of a straight line through $\mathbf{r}_3 = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{r}_4 = 7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

   Show that the equation can be written in the form $\dfrac{x-2}{5} = \dfrac{y-1}{-3} = \dfrac{z+3}{7}$.

5. Show that $\dfrac{x+2}{2} = \dfrac{y+8}{5} = \dfrac{z+5}{3}$ and $\dfrac{x-10}{4} = \dfrac{y-22}{10} = \dfrac{z-13}{6}$ represent the same line.

6. **Exam April 2006.** (Slightly modified version ☺)
   Prove De Moivre's Theorem, that is, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.
   If $z_1 = 3 - 4i$ and $z_2 = -\sqrt{3} + i$,
   (i) find the real and imaginary parts of z_1^{-1};
   (ii) find the moduli and principal value of the arguments of z_1, z_2 and $\dfrac{z_2}{z_1}$;
   (iii) find the modulus and principal value of the argument of z_2^7;
   (iv) find the modulus and arguments of all values of $z_1^{\frac{1}{2}}$;
   (v) plot the results of (iv) in the complex plane.
   Now let z denote a complex number and consider the equation $z^8 = 1$.
   (vi) How many different solutions does this equation have?
   (vii) Find all the solutions to the equation and plot them in the complex plane.