Problems for Lecture 11: Matrices I

1. Find the matrix products:

(a) $\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}5\\6\end{pmatrix}$, (b) $\begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}5\\6\end{pmatrix}$, (c) $\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}5\\6\end{pmatrix}$, (d) $\begin{pmatrix}5\\6\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$,

(e) $\begin{pmatrix}1 & -3 & 5 & -7\end{pmatrix}\begin{pmatrix}8\\6\\4\\2\end{pmatrix}$, (f) $\begin{pmatrix}8\\6\\4\\2\end{pmatrix}\begin{pmatrix}1 & -3 & 5 & 7\end{pmatrix}$.

2. (a) Write down the 2×2 rotation matrices $\mathbf{R}_{+45^\circ}, \mathbf{R}_{+90^\circ}$, representing 45° and 90° *anti-clockwise* (*counter-clockwise)* rotations, respectively.

(b) Show that the product of two 45° *anti-clockwise* rotation matrices is equivalent to a single 90° *anti-clockwise* rotation matrix, that is, $\mathbf{R}_{+45^\circ}\mathbf{R}_{+45^\circ} = \mathbf{R}_{+90^\circ}$.

(c) What does the product $\mathbf{R}_{+90^\circ}\mathbf{R}_{+90^\circ} = \mathbf{R}^2_{+90^\circ}$ of two 90° *anti-clockwise* rotation matrices correspond to? What does the product $\mathbf{R}_{-90^\circ}\mathbf{R}_{-90^\circ} = \mathbf{R}^2_{-90^\circ}$ of two 90° *clockwise* rotation matrices correspond to?

3. (a) Write down the 2×2 matrix $\mathbf{R}_{-|\theta|}$ representing a *clockwise* rotation of axes by an angle of $\theta = \sin^{-1}(4/5)$. Express θ in degrees.

(b) If the *clockwise* rotation in 3(a) is followed by an *anti-clockwise* rotation of 45°, find the matrix $\mathbf{R}_{\mathbf{net}}$ representing the net rotation. From the nature of the matrix, deduce whether the net rotation is clockwise or anti-clockwise.

(c) Show that the order of the two axes rotations in parts (a) and (b) is irrelevant.

(d) Deduce the inverse of the matrix $\mathbf{R}_{\mathbf{net}}$.

4. Evaluate the determinants (a) $\det \mathbf{A} = \begin{vmatrix}4 & 0 & 0 & 0\\6 & -1 & 0 & 0\\5 & 4 & 3 & 0\\3 & 2 & 1 & 2\end{vmatrix}$, (b) $\det \mathbf{B} = \begin{vmatrix}4 & 0 & 1 & 0\\6 & -1 & 0 & 0\\5 & 4 & 3 & 0\\3 & 2 & 1 & 2\end{vmatrix}$.