# *Problems for Lecture 10: Matrices I*

In questions 1-3, the $2\times 2$ matrices $\mathbf{A},\mathbf{B}$ and $\mathbf{C}$ are

$$\mathbf{A}=\begin{pmatrix}2 & 3\\ 4 & 6\end{pmatrix},\quad \mathbf{B}=\begin{pmatrix}2 & -1\\ 5 & 4\end{pmatrix},\quad \mathbf{C}=\begin{pmatrix}-4 & 2\\ 0 & 1\end{pmatrix}.$$

1. Find the matrices given by (a) $3\mathbf{A}$, (b) $\mathbf{A}+\mathbf{B}$, (c) $3\mathbf{B}-2\mathbf{C}$.

2. If a column vector ($2\times 1$ matrix) is represented by $\mathbf{r}=\begin{pmatrix}3\\ 4\end{pmatrix}$,

   find (a) $\mathbf{r}_1=\mathbf{A}\mathbf{r}$, (b) $\mathbf{r}_2=\mathbf{B}\mathbf{r}$, (c) $\mathbf{r}_3=\mathbf{r}_1+\mathbf{r}_2$. (d) $\mathbf{r}_4=(\mathbf{A}+\mathbf{B})\mathbf{r}$.

   Explain the property that implies that $\mathbf{r}_3=\mathbf{r}_4$. Prove that property in general for $2\times 2$ matrices and $\mathbf{r}=\begin{pmatrix}x\\ y\end{pmatrix}$.

3. Find the matrix products:

   (a) $\mathbf{AB}$, (b) $\mathbf{BC}$, (c) $\mathbf{CB}$, (d) $\mathbf{AC}$,

   (e) $(\mathbf{AB})\mathbf{C}$, (f) $\mathbf{A}(\mathbf{BC})$, (g) $(\mathbf{A}+\mathbf{B})\mathbf{C}$, (h) $\mathbf{AC}+\mathbf{BC}$.

4. If the matrices $\mathbf{P}=\begin{pmatrix}2 & 3 & 1 & -4\\ 2 & 1 & 0 & 5\end{pmatrix}$, $\mathbf{Q}=\begin{pmatrix}2 & 4\\ 1 & -1\\ 3 & -1\end{pmatrix}$, $\mathbf{R}=\begin{pmatrix}2 & 1 & 3\\ 4 & -1 & -2\\ -1 & 0 & 1\end{pmatrix}$, either

   calculate, or discard as meaningless, all nine potential products of two of the matrices.

5. Find the inverse $\mathbf{A}^{-1}$ of the $2\times 2$ matrix $\mathbf{A}=\begin{pmatrix}9 & 6\\ 5 & 3\end{pmatrix}$ using “brut force” calculation.

6. Find the vector resulting from the counter-clockwise rotation of $\mathbf{r}=\begin{pmatrix}3\\ 4\end{pmatrix}$ in question 2 by $60^\circ$ and $90^\circ$, respectively.

7. If $\mathbf{M}=\begin{pmatrix}4 & 1\\ 3 & 2\end{pmatrix}$, evaluate (a) $3\mathbf{M}$, (b) $\det\mathbf{M}$, (c) $\det(3\mathbf{M})$.

   Show in general that, if $\mathbf{M}$ is an $n\times n$ matrix, then $\det(r\mathbf{M})=r^n\det\mathbf{M}$, where $r$ is a factor. (Reminder: When a matrix is multiplied by a factor $r$, all elements of the matrix are multiplied by the factor $r$, i.e., , if $m_{ij}$ is the $ij$ th element of the matrix $\mathbf{M}$, then the $ij$ th element of the matrix $r\mathbf{M}$ is $rm_{ij}$.)