Appendix B

Hyperbolic Functions

The hyperbolic functions are defined as

$$\cosh x = \frac{\exp(x) + \exp(-x)}{2}, \tag{B.1a}$$

$$\sinh x = \frac{\exp(x) - \exp(-x)}{2}, \tag{B.1b}$$

$$\tanh x = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}, \tag{B.1c}$$

$$\coth x = \frac{\exp(x) + \exp(-x)}{\exp(x) - \exp(-x)} \quad \text{for } x \neq 0, \tag{B.1d}$$

$$\operatorname{sech} x = \frac{2}{\exp(x) + \exp(-x)}, \tag{B.1e}$$

$$\operatorname{csch} x = \frac{2}{\exp(x) - \exp(-x)} \quad \text{for } x \neq 0. \tag{B.1f}$$

The graphs of the hyperbolic functions are shown in Figure B.1.

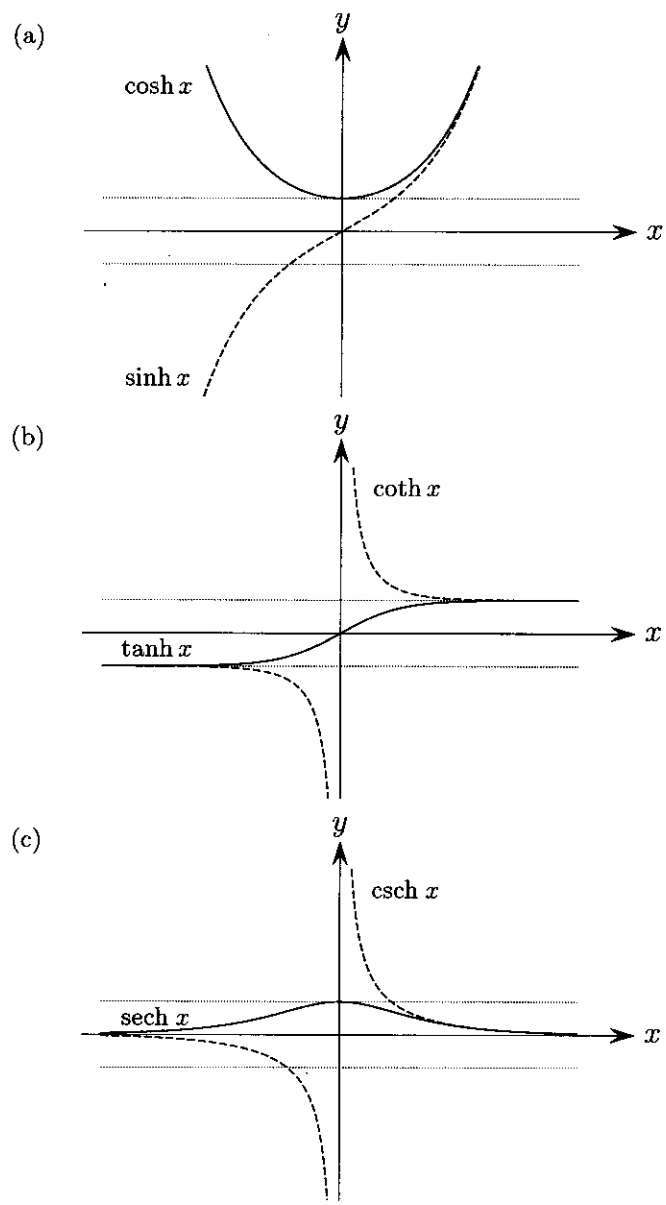

Fig. B.1 (a) Hyperbolic cosine (solid line) and sine (dashed line) of x. (b) Hyperbolic tangent (solid line) and cotangent (dashed lines) of x. (c) Hyperbolic secant (solid line) and cosecant (dashed lines) of x. The horizontal dotted lines are $y = 1$ and $y = -1$.