Mathematical Techniques II: Algebra (Physics Year 1 Term 1) **CW3 05/11/2007**

Classwork 3 – Air Traffic Control

In this Classwork, vectors are used to solve an air traffic control problem. A right-handed three-dimensional Cartesian coordinate system is used. The origin is a radio beacon and all distances are in units of km.

1. Given that the positive direction of the *x*-axis east and that the positive direction of the *y*-axis is north, what is the positive direction of the *z*-axis?

Let $\mathbf{i}, \mathbf{j}, \mathbf{k}$ denote the natural basis of $\mathbb{R}^3$ and let $\mathbf{r}_i = (x_i, y_i, z_i)$ for $i = 1, 2$.

- Flight PH01 is flying *level* at an altitude of 5km following the flight path $\mathbf{r}_1 = \mathbf{r}_{01} + \lambda \mathbf{d}_{01}$, $\lambda \in \mathbb{R}$, where $\mathbf{r}_{01} = -20\mathbf{i} + 20\mathbf{j} + 5\mathbf{k}$ and $\mathbf{d}_{01} = \mathbf{i} + 2\mathbf{j}$.
- Flight PH02 is *descending* on the flight path $\mathbf{r}_2 = \mathbf{r}_{02} + \mu \mathbf{d}_{02}$, $\mu \in \mathbb{R}$, where $\mathbf{r}_{02} = 5\mathbf{i} + 5\mathbf{j} + 7.2\mathbf{k}$ and $\mathbf{d}_{02} = -\mathbf{i} + \mathbf{j} - 0.1\mathbf{k}$.

Ignore the z dimension (altitude) in question 2, and just consider the flight paths as lines on a map or chart.

2. (a) Show that the path of flight PH01 is given by $x_1 + 20 = \dfrac{y_1 - 20}{2}$, and that of flight PH02 by $5 - x_2 = y_2 - 5$.

   (b) Plot the flight paths on a (two-dimensional) map, and find the coordinates x_0, y_0 of the point where they cross.

   (c) Find the angle θ between the two flight paths on your map.

   (d) Find the distance $p_i, i = 1, 2$ from the radio beacon to the closest point on each flight path.

Now include z dimension (altitude) and consider the flight paths of the aircrafts in 3D.

3. (a) Find the angle of descent δ of flight PH02.

   (b) Find the vertical separation of the two flight paths at the point where they cross when projected onto 2D.

   (c) If you were the air traffic controller and the planes were expected to arrive at the crossing point at roughly the same time, what would you do?

   (d) Find the nearest distance of each aircraft to the beacon at the origin. This is similar to question 2(d), but altitude is now involved. In the case of PH02, the downward slant of the flight path (which affects the distance very slightly) should be ignored.

   (e) Find the angle between the two flight paths in 3D and compare with your answer to question 2(c).