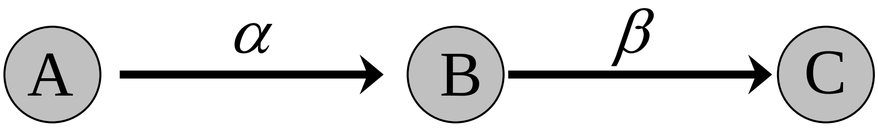
Classwork 1 – Decay

The Classwork is about sequential decay, that is, two decays in sequence. It has direct application to radioactive decay (e.g., ^{226}Ra decays to ^{222}Rn which decays to ^{218}Po), but there are other applications as well. The early sections of the Classwork 1 are relatively easy, but things get progressively harder as you go on. Ask a demonstrator if you get stuck. They are there to help you!

The diagram shows species A decaying into species B at rate α, and species B decaying into species C at rate β.



For parts (a)-(d), ignore the second decay and just focus on A decaying into B.

(a) Using the symbol $A(t)$ for the population of species A, write down the differential equation governing its decay into species B, and show that the solution is

$$A(t) = A(0)e^{-\alpha t}.$$

(b) What is the SI-units of α? What is the units of αt?

(c) In terms of α, find the times t at which (i) $A(t) = A(0)e^{-1}$ and (ii) $A(t) = A(0)/2$.

(d) Write down the corresponding expression for the population $B(t)$ of species B in terms of $A(0)$ and $B(0)$.

Now tackle the more full problem in which species B also decays into species C at rate β.

(e) Write down the differential equation for $B(t)$ in the new situation, and solve it to show that

$$B(t) = B(0)e^{-\beta t} + \alpha A(0)\frac{(e^{-\alpha t} - e^{-\beta t})}{\beta - \alpha}$$

(f) In the case where $B(0) = 0$, study the behaviour of $B(t)$ in the limits when (i) $\alpha \gg \beta$, and (ii) $\alpha \ll \beta$. In each case, explain what is happening in words, and draw a rough graph of $B(t)$ vs. t assuming for simplicity $A(0) = 1$.

(g) Find the solution of $B(t)$ when $B(0) = 0$ and $\alpha = \beta$.