Problem sheet 4, January 2005

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1. A simple RC high pass filter is constructed from a 10 kΩ resistor and 2 μF capacitor. Draw an appropriate circuit to realise this function and derive the transfer function $\left(\tilde{V}_o/\tilde{V}_i\right)$. Identify the 3dB frequency and sketch a Bode plot for the circuit.

   What is the output of the filter when fed an input voltage $V_i(t) = 10 \cos(70\, t) +15$ V.

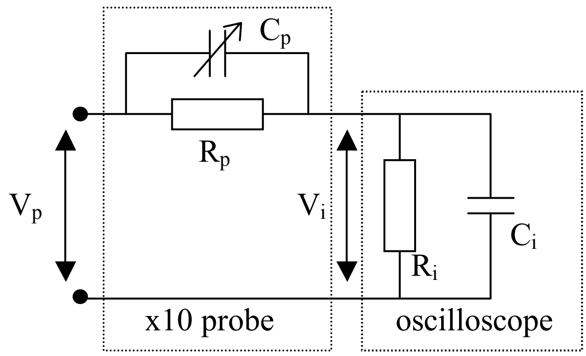
2. A power supply designed to produce a constant 5V DC output also produces a small amount of high frequency "noise" at 100 kHz. Design an RC filter to remove 99.9% of this noise without affecting the 5V output and keeping the output impedance of the power supply below 1Ω.

3. The input to particular oscilloscope is stated to be equivalent to a 1MΩ resistor in parallel with a 13pF capacitor. The oscilloscope is used to monitor an experimental setup, which has a Thevenin equivalent of a 1V source in series with a 10kΩ resistor.

   Draw a circuit representing the experimental setup when connected to the oscilloscope and determine what voltage is actually seen at the input to the oscilloscope as a function of frequency.

   What is the 3dB frequency of this arrangement?

4. A x10 oscilloscope probe designed to be used with a R_i=1MΩ input impedance oscilloscope (which also has a parallel input capacitance C_i) usually consists of a R_p=9MΩ resistor in parallel with a small variable capacitor C_p. Show that this circuit results in a voltage at the oscilloscope of $V_i=V_p/10$ when $R_pC_p=R_iC_i$.



   If C_i=13pF what is the required value of C_p?

   At high frequencies (>MHz), show that the impedances of the capacitors is significantly smaller than that of the resistors and hence that the probe can be assumed to be equivalent to a combined capacitance of C_i in series with C_p. Thus show that the input capacitance of the probe and oscilloscope combined is $C_i/10$=1.3pF. What does this do to the 3dB frequency when the x10 probe is used to look at the experiment in question 3.

   [You should look at the effect of C_p on a x10 probe next time you use one in the lab by connecting the probe to the oscilloscope square wave test signal and varying C_p]

5. Derive from first principles an expression for the impedance of a series LCR circuit as a function of angular frequency ω in complex exponential form Z and find |Z| and Arg[Z]. Use this expression to find the resonant frequency of the circuit, and the Q factor.

   Find the current in the RCL loop and hence the voltage across the resistor as a function of frequency.

6. The voltage, $\tilde{V}_C$, across the capacitor in a series LCR circuit driven by a voltage source $\tilde{V}_i$ is given by:

$$\frac{\tilde{V}_C}{\tilde{V}_i} = \frac{1}{1 + j\omega RC - \omega^2 LC} = \frac{1}{1 + j\dfrac{\omega}{\omega_o Q} - \dfrac{\omega^2}{\omega_o^{\,2}} LC}$$

   Depending on the exact value of Q this transfer function will have a peak in it at some frequency, which in general will not be at the resonant frequency. By differentiating the above equation show that the peak angular frequency at the peak is given by:

$$\omega_p = \omega_o \sqrt{1 - \frac{1}{2Q^2}}$$

   [Hint: Calculate when the differential $d\left|\frac{\tilde{V}_C}{\tilde{V}_i}\right|^2 \Big/ d\left(\frac{\omega^2}{\omega_o^{\,2}}\right)$=0 to find the peak]

   Thus show that there is no resonant peak when Q<0.5. And find the value of R required to achieve this when L=4.7mH and C=100pF.

7. A band pass filter with is required with a centre frequency of 1 MHz and FWHM bandwidth of 10 KHz. Approximately what value of Q is required to meet these requirements?

   Show that a resistor in series with a parallel combination of capacitor and inductor can fulfil these requirements and suggest suitable component values.

8. Derive from first principles a differential equation in terms of charge q that describes the voltage in an LCR series circuit containing a sinusoidal voltage source V(t) = V_o cos (ωt). Explain how this equation relates to that for a simple mechanical forced harmonic oscillator?

9. I can be shown that the natural response of an undriven, underdamped LCR resonant circuit is given by:

$$V(t) = V_o e^{-\alpha t}\cos(\omega' t + \phi)$$

$$\alpha = \frac{\omega_o}{2Q}; \omega' = \omega_o \sqrt{1 - \frac{1}{4Q^2}}$$

   Explain what factors affect V_o and ϕ for any particular situation.

   For both series and parallel LCR resonant circuits write down ω_o and Q in terms of the component values, and hence determine α and ω' in terms of the circuit component values.

   Sketch a typical graph of V(t) when Q=10.