Electronics problem sheet 3 - solutions, January 2005

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1.

Resistors	$V = IR$	
Inductors	$V = L\dfrac{dI}{dt}$	$I = \dfrac{1}{L}\int Vdt$
Capacitors	$V = \dfrac{1}{C}\int Idt$	$I = C\dfrac{dV}{dt}$

2.

$$C = \frac{\varepsilon_r \varepsilon_o A}{d}$$

$$A = \frac{Cd}{\varepsilon_o} = \frac{10^{-6} \times 10^{-3}}{8.85 \times 10^{-12}} = 113m^2$$

If breakdown voltage is 1kV/mm then maximum voltage allowed is 1kV. Energy stored $=0.5CV^2=0.5\text{x}10^{-6}\text{x}(10^3)^2=0.5$J.

3.

$$L = \frac{\mu N^2 A}{l}$$

$$N = \sqrt{\frac{lL}{\mu_r \mu_o A}} = \sqrt{\frac{0.05 \times 10^{-4}}{400 \times 4\pi \times 10^{-7} \times \pi \times 0.004^2}} = 14.06$$

14 turns is closest value

4. At DC the inductor looks like a short circuit and so there can be no voltage across L. There is therefore no current through R_2 and the whole of the source voltage V falls across R_1. The current through R_1 is therefore $I_o=V/R_1$. This current then all flows through L too.

   At the instant the switch is opened there can be no current in R_1 as it is open circuited and hence the voltag across R_1 is 0V. Howver the full value of I_o must still be flowing down L and hence up R_2 (as drawn in the question) i.e. in a loop through L and R_2. The voltage across the inductor and the resistor R_2 must have identical magnitude (I_oR_2), but of opposite sign in respect to the direction of current flow as they form a loop where the total voltage around the loop must be zero. The sign is defined by the fact the top of the resistor and inductor is at a negative voltage relative to the bottom as drawn. Applying Kirchhoff's voltage law around this loop:

$$L\frac{dI}{dt}+IR_2=0 \qquad \frac{dI}{dt}=-\frac{R_2}{L}I$$

$$\int_{I_o}^{I}\frac{1}{I}dI=\int_{0}^{t}-\frac{R}{L}dt$$

$$\left[\ln(I)\right]_{I_o}^{I}=\ln\left(\frac{I}{I_o}\right)=\left[-\frac{R}{L}t\right]_0^t=-\frac{R}{L}t$$

$$I(t)=I_oe^{-\frac{R_2}{L}t}$$

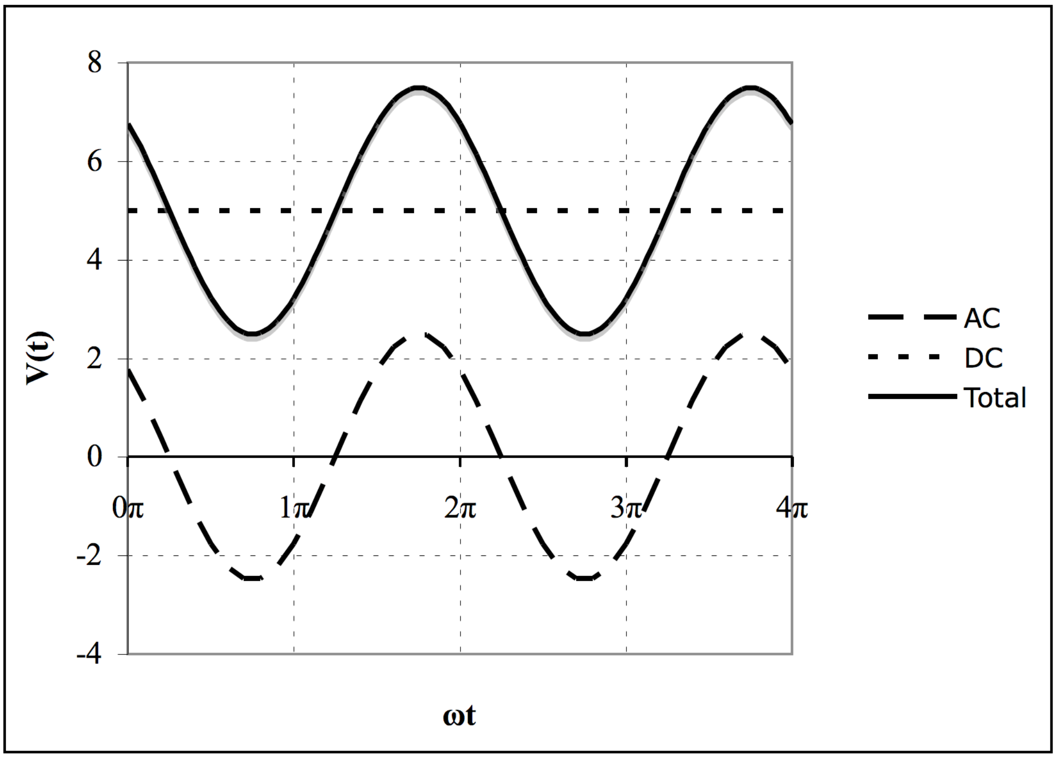
Instantaneous power in the resistor is given by P(t)=I^2(t)R and integrating power with respect to time gives the total energy, U, dissipated in the resistor (and which originally was stored in the inductor):

$$P(t)=I(t)^2R_2=I_o^{\,2}R_2e^{-\frac{2R_2}{L}t}$$

$$U=\int_0^{\infty}P(t)dt=\int_0^{\infty}I_o^{\,2}R_2e^{-\frac{2R_2}{L}t}dt=\left[-\frac{L}{2R_2}I_o^{\,2}R_2e^{-\frac{2R_2}{L}t}\right]_0^{\infty}=\frac{1}{2}LI_0^{\,2}$$

5. See graph below labelling DC and AC components for the required signal

   V(t) = 5 + 2.5 cos (ωt + π/4).



6. Assume *circular* contact capacitor electrodes of diameter 10mm.

   (i)

$$C=\frac{\varepsilon_r\varepsilon_oA}{d}=\frac{80\times8.85\times10^{-12}\times\pi\left(5\times10^{-3}\right)^2}{10^{-5}}=5.56\times10^{-9}F=5.6nF$$

(ii)

$$|Z| = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^9 \times 5.6 \times 10^{-9}} = 0.0286\Omega = 29m\Omega$$

Note that the capacitor looks very much like a short circuit (the peak current is actually $V_{max}/|Z|$=35A!!)

(iii) Maximum charge is stored when voltage is a maximum so:

$$Q_{\max} = CV_{\max} = 5.6 \times 10^{-9} \times 1 = 5.6nC$$

7.

(i)

$$V(t) = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\tilde{V} = 7e^{j\frac{\pi}{2}} = 7j$$

(ii)

$$V(t) = 3\sin(\omega t) = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

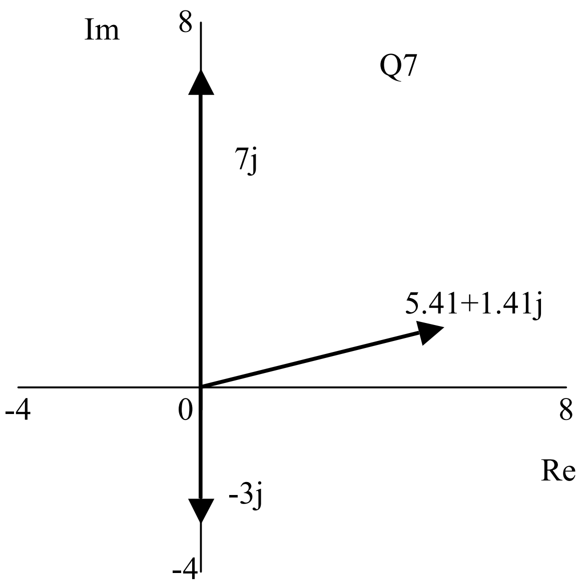
$$\tilde{V} = 3e^{-j\frac{\pi}{2}} = -3j$$

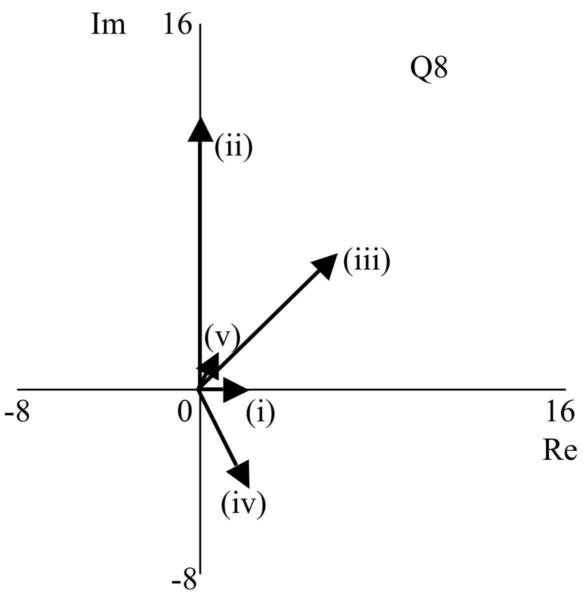
(iii)

$$V_1(t) = 2\cos\left(\omega t + \frac{\pi}{4}\right) \qquad \tilde{V}_1 = 2e^{j\frac{\pi}{4}} = \sqrt{2}(1+j)$$

$$V_2(t) = 4\sin\left(\omega t + \frac{\pi}{2}\right) = 4\cos(\omega t) \qquad \tilde{V}_2 = 4e^{j0} = 4$$

$$\tilde{V} = \tilde{V}_1 + \tilde{V}_2 = \left(4 + \sqrt{2}\right) + \sqrt{2}j = 5.60e^{j0.255}$$





8. See above for Argand diagram

(i) $\tilde{V} = 2 = 2e^{j0}$ $\quad V(t) = 2\cos(\omega t)$

(ii) $\tilde{V} = 12j = 12e^{j\frac{\pi}{2}}$ $\quad V(t) = 12\cos\left(\omega t + \frac{\pi}{2}\right) = -12\sin(\omega t)$

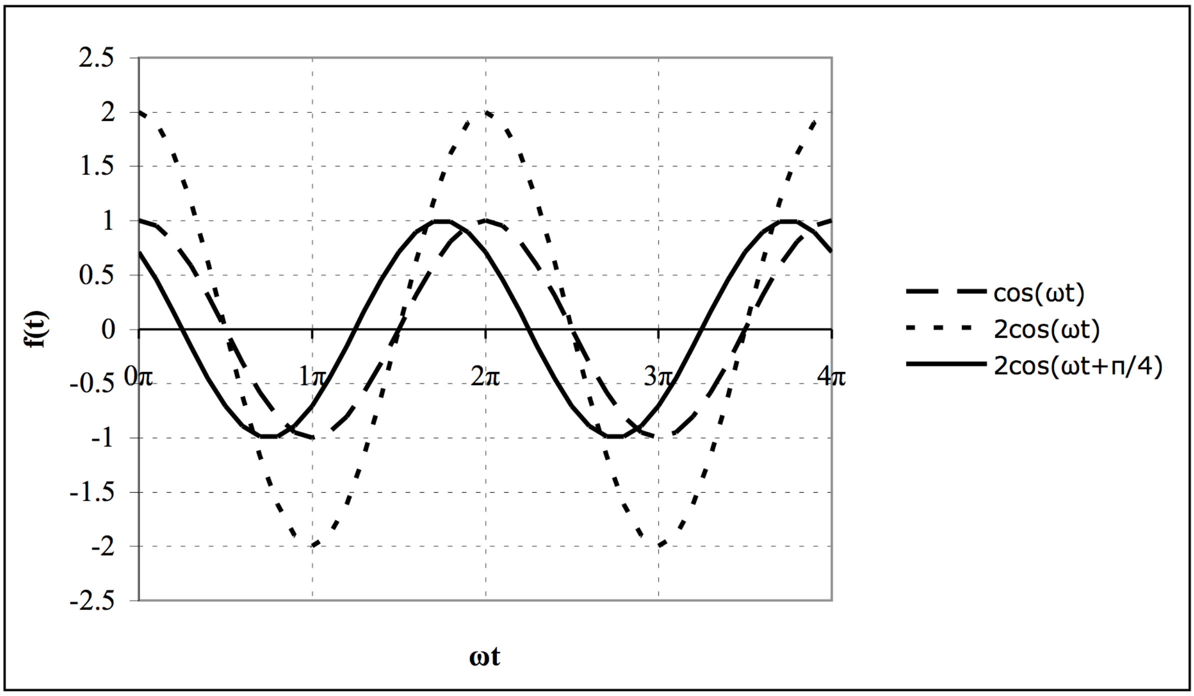
(iii) $\tilde{V} = 6 + 6j = 6\sqrt{2}e^{j\frac{\pi}{4}}$ $\quad V(t) = 6\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right) = 8.49\cos\left(\omega t + \frac{\pi}{4}\right)$

(iv) $\tilde{V} = 2 - 4j = \sqrt{2^2 + 4^2}e^{j\tan^{-1}\left(\frac{-4}{2}\right)} = \sqrt{20}e^{-j1.11}$

$V(t) = \sqrt{20}\cos(\omega t - 1.11) = 4.47\cos(\omega t - 1.11)$

(v) $\tilde{V} = 2e^{j\frac{\pi}{3}} = 1 + \sqrt{3}j$ $\quad V(t) = 2\cos\left(\omega t + \frac{\pi}{3}\right)$

9.



10.

$$\tilde{V} = V_o e^{j\frac{\pi}{4}}$$

$$\tilde{I} = \frac{V_o}{2}e^{j0}$$

$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{V_o e^{j\frac{\pi}{4}}}{\frac{V_o}{2}e^{j0}} = 2e^{j\frac{\pi}{4}} = \sqrt{2}(1+j)\Omega$$

Equivalent to a resistor and inductor in series where the magnitude of the impedance of the inductor equals that of the resistor (√2Ω).