Problem sheet 3, January 2005

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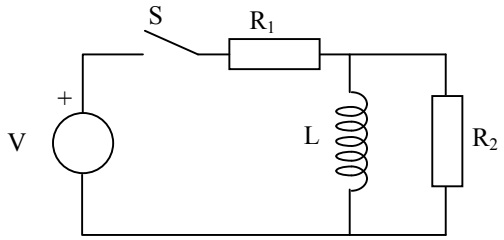
1. State Ohm's law for resistors, capacitors and inductors in a form that is valid for arbitrary voltage or current. For capacitors and inductors include both integral and differential relationships between current and voltage.
2. State the parallel plate capacitor equation giving the capacitance C between two parallel plates of area A and separation d with a dielectric filling of relative permittivity ε_r.

   You are attempting to construct your own air spaced parallel plate capacitor. For a plate separation of d=1mm, what area A is required to produce a 1μF capacitor.

   If the breakdown voltage of air is 1kVmm^{-1}, what is the maximum energy that this capacitor can hold?
3. The inductance of a long solenoid of length l, cross-sectional area A, with N turns and filled with a material of permeability μ is given by the equation:

$$L = \frac{\mu N^2 A}{l}$$

   A radio aerial is constructed using a long solenoid constructed by winding copper wire on a ferrite rod of relative permeability μ_r=400. The length of the rod is 5cm and it has a radius of 4mm. How many turns are required to produce a 100μH inductance.
4. In the following circuit the switch S is initially closed. After some time when the circuit has settled down to a steady state, what currents are flowing in the two resistors and the inductor.



   The switch S is now opened. At this instant what currents are flowing in each of the resistors and the inductor and what is the voltage across each element.

   Write down a differential equation describing the current flowing in the circuit and solve it to find the evolution of the current in the inductor as a function of time after the switch is opened.

   From this current find the power dissipated in the resistor R_2 at any time t after the switch is opened. By integrating this power show that the energy stored in the inductor is given by $\frac{1}{2}LI^2$ where I was the current flowing in the inductor when the switch was opened.

5. Describe what is meant by a DC voltage and an AC voltage with the aid of simple diagrams. Sketch the voltage $V(t) = 5 + 2.5 \cos(\omega t + \phi_o)$ for $\phi_o = \pi/4$.

6. Two circular contacts 10mm in diameter form the two plates of a capacitor and are separated by a thin, high dielectric constant layer 10^{-2} mm thick with $\varepsilon_r = 80$. A signal generator applies a pure sine wave with a frequency of 1 GHz and ±1V peak amplitude across the capacitor. Find (i) the capacitance (ii) magnitude of the impedance and (iii) peak charge stored in the capacitor under these conditions

7. Express the following voltages in complex exponential form $\tilde{V} = V_o e^{j\phi}$. Illustrate your answers on an Argand diagram.

   (i) $7 \cos(\omega t + \pi/2)$ (ii) $3 \sin(\omega t)$ (iii) $2\cos(\omega t + \pi/4) + 4 \sin(\omega t + \pi/2)$

8. Draw the following complex phasors on an Argand diagram and express them as time domain sinusoids

   (i) $\tilde{V} = 2$ (ii) $\tilde{V} = 12j$ (iii) $\tilde{V} = 6 + 6j$ (iv) $\tilde{V} = 2\text{-}4j$ (v) $\tilde{V} = 2e^{j\pi/3}$

9. Make a simple sketch of two time varying sinusoids that are different in amplitude by a factor of two but are in phase. Repeat this for two sinusoids that have the same amplitude but are out of phase by a fixed phase angle of $\pi/4$.

10. A voltage $V(t) = V_o\cos(\omega t + \pi/4)$ is applied across a circuit element and results in a current flow given by $I(t) = V_o/2 \cos(\omega t + 0)$. Find an expression for the complex impedance Z of the circuit element. Hint, express the voltage and current in complex exponential form and use complex Ohm's law.