Electronics problem sheet 2 - solutions, January 2005

Dr Mark Neil

1. Voltage difference across the resistor is 10-4=6V.

   So the current through the resistor is I=6/100=0.06A=60mA. Power in the resistor can be calculated in several ways: e.g. I^2R =0.06*0.06*100=0.36W=360mW.

   Increasing the voltage at both ends of the resistor by the same amount doesn't chance the voltage difference across the resistor so the current and power remain the same.

2.
   (a) Potential divider: Open circuit voltage $V_T=VR_2/(R_1+R_2+R_3)$. R_o is resistance between terminals with V replaced by short circuit = $R_2//(R_1+R_3)=R_2(R_1+R_3)/(R_1+R_2+R_3)$. $I_N=V_T/R_o=V/(R_1+R_3)$.

   (b) R_1 and R_2 in parallel gives impedance $R_4=R_1R_2/(R_1+R_2)$. Now looks like potential divider again:

   Open circuit voltage $V_T= VR_3/(R_3+R_4)= VR_3/(R_3+ R_1R_2/(R_1+R_2))$

   $$V_T=VR_3(R_1+R_2)/(R_1R_2+R_2R_3+R_3R_1)$$

   R_o is resistance between terminals if V is replaced by short circuit

   $$R_o=R_1//R_2//R_3= R_1R_2R_3/(R_1R_2+R_2R_3+R_3R_1)$$

   $I_N=V_R/R_o=V(R_1+R_2)/(R_1R_2)$

   (c) Two 2Ω resistors in parallel are equivalent to a single 1Ω resistor. 2A current source in parallel with 1Ω resistor is equivalent to a 2V source in series with 1Ω resistor. Adding 2V source in series with 3V source gives a total 5V source still in series with a 1Ω resistor. Circuit now looks like a simple potential divider:

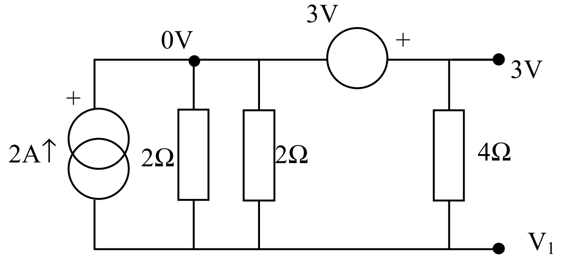
   $$V_T=5V\text{x}4\Omega/(1\Omega+4\Omega)=4V.$$

   $$I_N=I_{SC}=5V/1\Omega=5A.$$

   $$R_o=V_T/I_N=4/5\Omega$$

3. See notes "kirchhoff-nodal-mesh.pdf" on teaching website.

4.



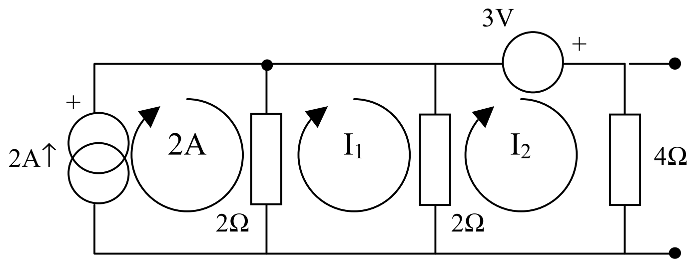
Choose reference node (0V) at bottom of voltage source and top of current source. Other nodes are then 3V and unknown V_1.

Single node voltage equation summing currents into V_1 gives:

$$-2A+\frac{(3-V_1)}{4}+\frac{(0-V_1)}{2}+\frac{(0-V_1)}{2}=0$$
$$-8+3-V_1-2V_1-2V_1=0$$
$$5V_1=-5$$
$$V_1=-1V$$

Check V_{oc}=3-V_1=4V (same as answer to 2(c))

5.



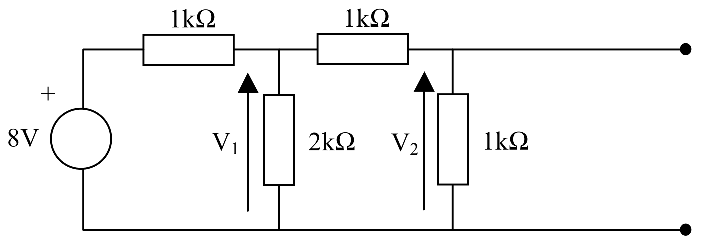
Three loops as shown. One loop has known current 2A flowing round it, the other two have unknown currents I_1 and I_2 flowing round them. Sum voltages round the loops I_1 and I_2:

$$(I_1-2)\times 2+(I_1-I_2)\times 2=0$$
$$4I_1-2I_2-4=0$$
$$(I_2-I_1)\times 2+(I_2)\times 4=3$$
$$6I_2-2I_1=3$$
$$6I_2-(I_2+2)=3$$
$$5I_2=5$$
$$I_2=1A$$
$$I_1=(4+2I_2)/4=1.5A$$

Current down 2Ω resistors = 2-I_1=0.5A or I_1-I_2=0.5A (both should be the same anyway). Current down 4Ω resistor=I_2=1A.

6. The relationship between $V_2$and V_3 can be easily obtained at the two 1kΩ resistors form a potential divider such that V_3=V_2/2.

   These two 1kΩ resistors also form a combined 2kΩ resistance when added in series. This 2kΩ resistance lies in parallel with the 2kΩ resistance across which V_2 is measured and so the pair of them can be replaced by a single 1kΩ resistor as shown below.



   Thus the relationship between V_2 and V_1 becomes apparent as it is identical to that between V_3 and V_2, ie V_2=V_1/2.

By repeating the same resistor simplification we can also see that V_1=8V/2=4V.

Thus V_1=4V, V_2=2V and V_3=1V.

7. In all cases apply the virtual earth approximation $V_+=V_-$ and no current flows into the input.

   (a) $V_+=V_-=V_i$=1V. Summing currents into V_- and using R_2=9kΩ and R_1=1kΩ:

$$\frac{(V_o - V_-)}{R_2} + \frac{(0 - V_-)}{R_1} = 0$$
$$(V_o - V_i)R_1 - V_i R_2 = 0$$
$$V_o = V_i\left(1 + \frac{R_2}{R_1}\right) = 1V\left(1 + \frac{9k\Omega}{1k\Omega}\right) = 10V$$

   (b) $V_+=V_-$=0V. Summing currents into V_- and using R_2=200kΩ and R_1=10kΩ:

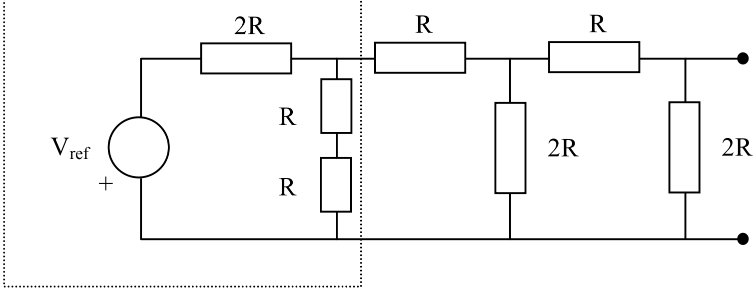
$$\frac{(V_o - V_-)}{R_2} + \frac{(V_i - V_-)}{R_1} = 0$$
$$V_o R_1 + V_i R_2 = 0$$
$$V_o = -V_i\left(\frac{R_2}{R_1}\right) = -10mV\left(\frac{200k\Omega}{10k\Omega}\right) = -200mV$$

   (c) $V_+=V_-$=0V. Summing currents into V_- and using R=1MΩ, I=5μA:

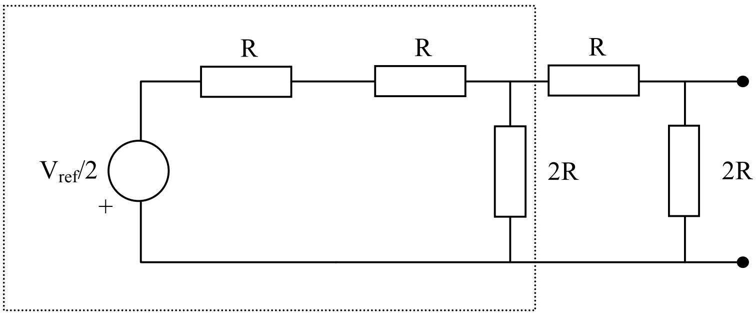
$$-I + \frac{(V_o - V_-)}{R_1} = 0$$
$$V_o - IR = 0$$
$$V_o = IR = 5\mu A \times 1M\Omega = 5V$$

8. With all switches connected to ground the Thevenin equivalent up to the op-amp input must have 0V as its voltage source. Switching in each of the S_n to $-V_{ref}$ individually will produce a Thevenin voltage source V_{Tn} which can be summed appropriately (by superposition) to give the total Thevenin voltage according to the combination of S_n.

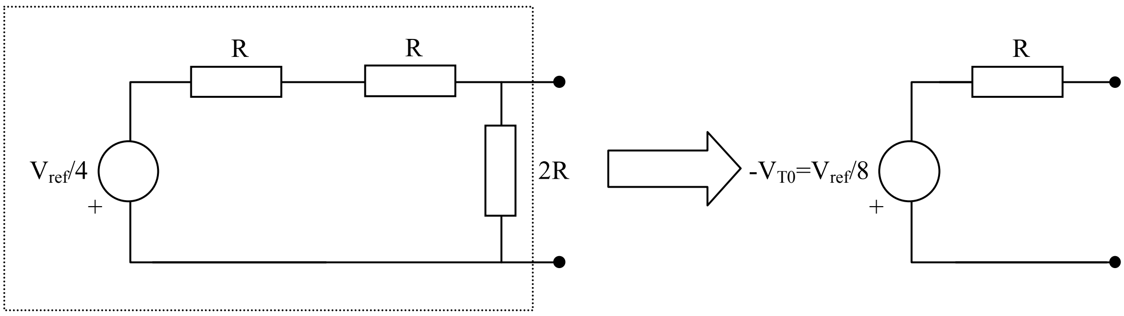
   First consider case of S_o closed to $-V_{ref}$ and S_1 and S_2 closed to ground (0V). A slight rearrangement of the circuit up to the input to the op-amp gives:



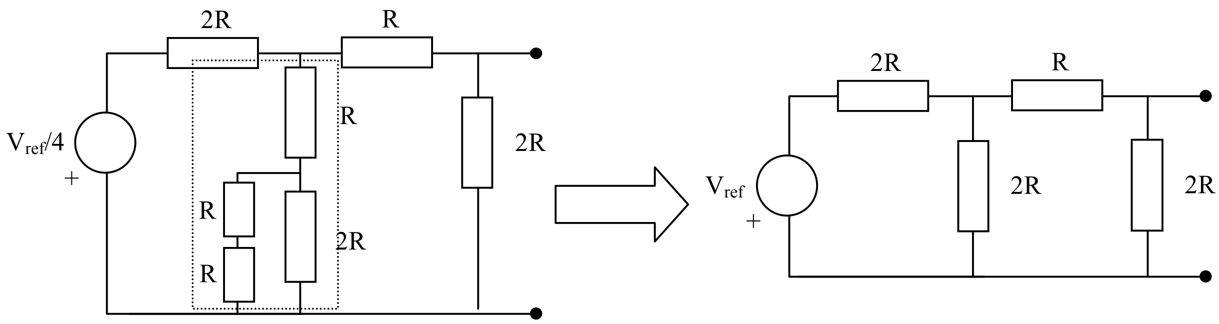
Replacing the boxed circuit with its Thevenin equivalent gives:



Repeating the process gives:



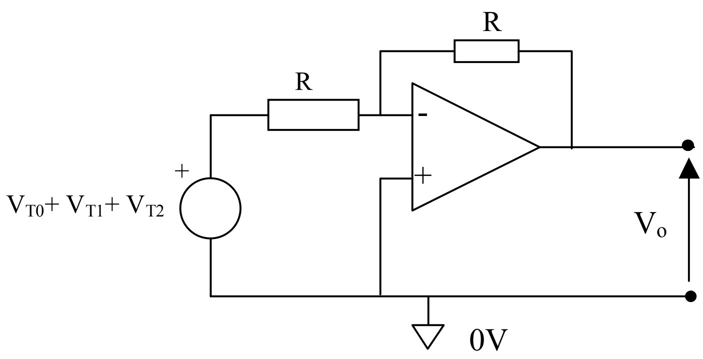
Now look at S_1 closed to $-V_{ref}$ and S_2 and S_3 closed to ground (0V). Rearrangement of the circuit and then simplifying the boxed network of resistors gives:



Now looks like case above at top of page but producing $V_{T1}=-V_{ref}/4$.

Similar approach for S_2 produces $V_{T2}=-V_{ref}/2$.

Summing all Thevenin voltages by superposition gives a circuit:



Circuit now forms inverting amplifier with gain -1. So $V_o=-(V_{T0}+V_{T1}+V_{T2}+)=7V_{ref}/8$.

Circuit can generate voltages up to $7V_{ref}/8$ in steps of $V_{ref}/8$ according to the positions of the various S_n.