# Problem sheet 1 - solutions, January 2005

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1. Charge is an inherent and indestructible property of certain forms of matter. It is measured in Coulombs. 1 coulomb is the charge of $1/(1.6\text{x}10^{-19})=6.25\text{x}10^{18}$ electrons.

   Current is the flow of charge. 1 amp of current is the flow of 1 coulomb of charge per second across a boundary.

   Voltage is a measure of the potential energy of charge possesses. Raising a charge of 1 coulomb by 1 volt increases its potential energy by 1 joule.

2. Assume that the computer contains 100g of copper. Copper has a density of about $8.9\text{gcm}^{-3}\approx 10\text{gcm}^{-3}$. This implies that the computer contains $100/10=10\text{cm}^3$ of copper.

   If 1 $\text{m}^3$ contains $7\text{x}10^{28}$ electrons then $10\text{cm}^3$ contains $7\text{x}10^{23}$ electrons. Total free electron charge in copper = $7\text{x}10^{23}\text{x}1.6\text{x}10^{-19} \approx 10^5\text{C}$.

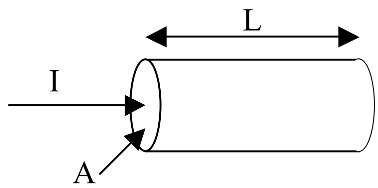
   Power from power supply=100W =VxI = 5xI so $I=20A=20Cs^{-1}$.

   Time to turn over charge in copper = $10^5/20$ = 5000 s $\approx$1.5 hours.

   Mircoprocessor takes 10A of current = $10Cs^{-1} = 10/(1.6\text{x}10^{-19})$ electrons per second $\approx 6\text{x}10^{19}$ e $s^{-1}$.

   Electrons per computation = $6\text{x}10^{19}/(10^9) = 6\text{x}10^{10}$ electrons.

3. Assume a cylindrical resistor geometry where the electric field and the current density are uniform and pointing along the axis of the cylinder:

   Resistance of cylinder, $R = L/(\sigma A)$

   Voltage along cylinder V=ExL

   Current density in the wire J=I/A.

   From ohm's law:

   $$V=IR$$

   Substitute for V, I and R:

   $$EL=JA \text{ x } L/(\sigma A)=JL/\sigma$$

   Rearrange and cancel L :

   $$J=\sigma E$$

   QED

4. Assume film thickness of d and that the film is sufficiently thin that we can write its cross sectional area as dx(cylinder circumference)$=3\pi d\text{x}10^{-3}$.

Resistance R is given by

$R=\rho L/A=8\rho x10^{-3}/(3dx10^{-3})=8\rho/(3\pi d)$.

Can calculate d from

$d=8\rho/(3\pi R)$.

put in values:

| | Copper | Carbon film | Metal oxide |
|---|---|---|---|
| 1Ω | 14.4 nm | 0.51mm | 8.5 mm |
| 1kΩ | 14.4 pm | 0.51μm | 8.5 μm |
| 1MΩ | 14.4 fm | 0.51 nm | 8.5 nm |

Realistic values are probably in range 1nm to 100μm - only a range of $10^5$ in resistance is possible. Need to be able to tailor material resistivity by varying composition (eg metals in the oxide mix).

For ceramic substrate

$R=\rho L/A==\rho x8x10^{-3}/(\pi x(3x10^{-3})^2/4)=32{,}000\rho/(9\pi)>10^8\Omega$

So

$\rho>8.8x10^5\Omega m$

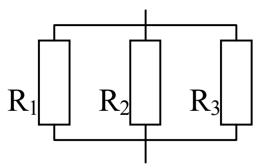
5. Symmetry determines that the current flows out from the centre in a radial direction. So hemispherical shells are on equipotentials. Consider a single shell of radius r and thickness dr. Its cross surface area is $2\pi r^2$ and hence its resistance dR is given by:

   $dR=\rho dr/(2\pi r^2)$

   The resistance of a series of shells (like layers on the onion) add in series so the total resistance can be obtained by integration.

$$R=\int_{r_o}^{\infty}\frac{\rho}{2\pi r^2}dr=\left[\frac{-\rho}{2\pi r}\right]_{r_o}^{\infty}=\frac{\rho}{2\pi r_o}=\frac{10}{2\pi 0.1}=16\Omega$$

6.




$$R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

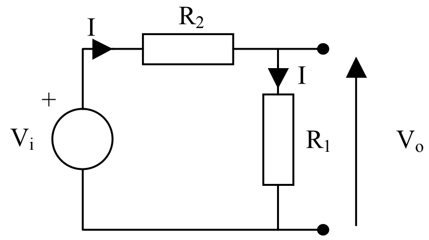
Multiply top and bottom by $R_1R_2R_3$ and simplify.

$$R=\frac{R_1R_2R_3}{\frac{R_1R_2R_3}{R_1}+\frac{R_1R_2R_3}{R_2}+\frac{R_1R_2R_3}{R_3}}=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$

Put in values:

$R=(2x3x6)/(3x6+2x6+2x3)=36/36=1\Omega$

7.




As no current is drawn and the output terminals the current in both resistors must be the same (I). By Kirchhoff's voltage law we can write

$V_i=IR_2+IR_1$

The output voltage $V_o$ is the voltage across $R_1$ so

$V_o=IR_1$

Taking the ratio of $V_o/V_i$ gives

$V_o/V_i=IR_1/(IR_1+IR_2)=R_1/(R_1+R_2)$

Or

$V_o=V_i\ R_1/(R_1+R_2)$

8. Energy of each photon is given by:

$E_{ph}=hc/\lambda$ [h=Planck's constant, c=speed of light , $\lambda$=wavelength]

So number of photons hitting photodiode per second N is given by:

$N=Power/E_{ph}=10^{-3}x632.8x10^{-9}/(6.63x10^{-34}x3x10^{8})=4.77x10^{15}\ s^{-1}$

Current generated $I = Ne = 4.77x10^{15}\ x1.6x10^{-19}$=0.51mA

Current source can't put out more power than it receives so VI<1mW so V<1.96V

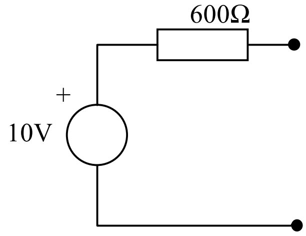
Energy of 632.8nm photon in $eV=hc/(\lambda e)$=1.96eV

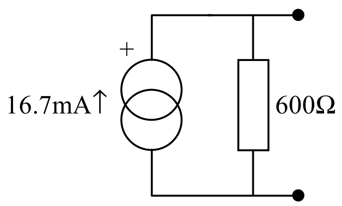
9. Assume knob is turned up to full 10V:

$V_{OC}=V_T$=10V

$R_o=600\Omega$ as stated

$I_N=V_T/R_o$=10/600=16.7mA



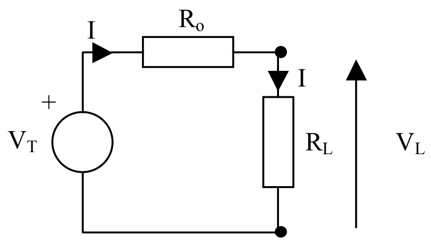




Input impedance of amplifier is 20kΩ so by a potential divider the voltage appearing at the output of the signal generator and the input to the amplifier is now

$V = 10x20/(20+0.6) = 9.709V$

Gain of amplifier is x100 so the voltage delivered to the load is 970.9V

10.




Power dissipated in the load $P_L$ is given by

$P_L = I^2 R_L$

Total power dissipated $P_T$ is given by:

$P_T = I^2(R_o + R_L)$

So efficiency = $P_L/P_T = R_L/(R_o+R_L) = 50\%$ When $R_o = R_L$

For 90% efficiency $R_L/(R_o+R_L) = 0.9$

$R_L = 0.9R_o/(1-0.9) = 9R_o$

Total current now flowing I

$I = V_T/(R_o+R_L) = V_T/(10R_o)$

Power in load $= I^2 R_L = 9R_o V_T^2/(100R_0^2) = 0.09V_T^2/R_o$

Compare with maximum power of

$P_{MAX} = 0.25V_T^2/R_o$

Only 36% of $P_{MAX}$