Problem sheet 1, January 2005

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1. Define the terms charge, current and voltage and the units in which they are measured - coulombs, amps and volts.
2. All the hard work inside a computer is done by electrons. Make a sensible estimate of how many conducting electrons there are in your average desktop computer [hint assume that the bulk of the electrons are sitting in copper wires with an electron density of $7 \times 10^{28} m^{-3}$ and make a sensible estimate of the quantity of copper inside the computer]

   The power supply in your average desktop computer is putting out about 100W into a voltage of 5V. From the current that this results in estimate roughly how often the electrons are recycled through the system?

   Looking just at the microprocessor alone you might typically see a 10A current consumption. Assuming that the microprocessor can perform about 10^9 computations per second, approximately how many electrons are involved in each computational step?
3. The current density $\underline{J}$ (a vector giving the magnitude and direction of current flow per unit area) to the local electric field $\underline{E}$ in a material are related via the material conductivity σ with the following equation:

   $$\underline{J} = \sigma \underline{E}$$

   People often refer to this equation as an alternative statement of Ohm's law. By considering a simple resistor geometry (such as a block or a cylinder) show that this equation is indeed an alternative statement of Ohm's law.
4. Real resistors are often constructed by depositing a thin film onto the curved surface of a cylindrical ceramic substrate and then forming electrical contacts at each end of the cylinder.

   Assuming that the cylinder has a diameter of 3mm and a length of 8mm estimate the thickness of film required to produce 1Ω, 1kΩ and 1MΩ resistors when the material used is:

   (a) Copper ($\rho = 1.7 \times 10^{-8}$ Ωm).

   (b) Carbon ($\rho = 6 \times 10^{-4}$ Ωm).

   (c) Metal-oxide film ($\rho = 10^{-2}$ Ωm).

   In each case comment on the feasibility of producing the desired resistance with these materials and comment on the importance of being able to tailor resistivity by varying the exact material composition.

   [Hint you can assume that the film thickness is significantly smaller than the cylinder diameter]

   What is the lower limit on the resistivity of the ceramic cylinder substrate material such that its resistance will be greater than 100MΩ.

5. A lightning conductor is connected to earth made by a 10cm radius conducting sphere half buried in the ground. If the resistivity of the ground is estimated to be 10Ωm, find the resistance to earth of this connection. [Hint think of the connection to earth as being a set of hemi-spherical shells (like layers of an onion) connected in series].

6. The combination of two resistors R_1 and R_2 in parallel can be replaced with a single resistor of value $R_1R_2/(R_1+R_2)$. Derive a similar expression for the combined resistance of three resistors R_1, R_2 and R_3 in parallel. Calculate this combined parallel resistance when $R_1=2\Omega$, $R_2=3\Omega$, $R_3=6\Omega$.

7. Describe how a pair of resistors can be used as a so-called potential divider to produce a scaled down copy of a voltage and derive the equation that gives the scaling factor.

8. Light can be detected in a device known as a photodiode where each incident photon generates an electron and a hole in the semiconductor. The electron can then effectively flow around an external circuit before recombining with the hole again. In this respect a photodiode behaves like a current source. Assuming a 1mW laser of wavelength 632.8nm is incident on the photodiode and that the quantum efficiency (conversion from photons to electrons) is 100%, what is the magnitude of the current produced by the photodiode?

   Calculate an upper limit on the voltage that can be produced at the terminals of the photodiode while still maintaining this constant current and satisfying energy conservation laws.

   Compare this to the energy of 632.8nm photons in electron-volts (eV)?

9. A signal generator in the lab has an output level knob with a scale reading up to 10V and has a label next to its output connector stating an output impedance of 600Ω. Draw the Thévenin equivalent for the output of this signal generator and hence deduce the Norton equivalent for this circuit.

   This signal generator is then fed into a high voltage amplifier to drive an experiment. The stated input impedance of the amplifier is 20kΩ and the gain is x100. What is the maximum voltage that can be delivered to the experiment by this combination of signal generator and amplifier.

10. Maximum power theorem states that the maximum power is extracted from an electrical circuit represented by a Thévenin or Norton equivalent when the load resistance, R_L, equals the output resistance, R_o, of the electrical circuit. Show that the efficiency (the ratio of power in the load to the total power dissipated in the combined circuit) is only 50%.

    There is often a conflict of interest between the desire to extract as much power from a circuit and the desire to do so as efficiently as possible. What load resistance would be required to increase the efficiency to 90%. And compare the amount of power delivered to the load relative to the maximum attainable when $R_L=R_o$.