Operational amplifiers

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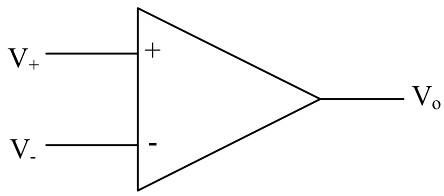
Operational amplifiers, or op-amps, are a very common circuit element, which although containing several tens of components in integrated circuit form can be thought of as a "black box" circuit element with range of useful applications. The black box is represented in its simplest form by the following circuit diagram element:



The connections shown are referred to as the non-inverting input (V_+), the inverting input (V_-) and the output (V_o). Note that this doesn't fully represent all the connections to the op-amp integrated circuit but is often all that is drawn in a circuit diagram. In practice the integrated circuit chip must also have +ve and –ve power supply connections (otherwise where do the currents come from and go to) and may have other connections (eg offset-null adjustment) to fine-tune its performance.

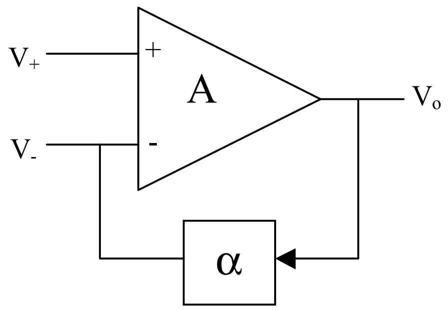
As drawn above the operational amplifier operates as an extremely high gain differential amplifier. That is, the output voltage V_o is an amplified version of the difference between the input voltages (V_+-V_-):

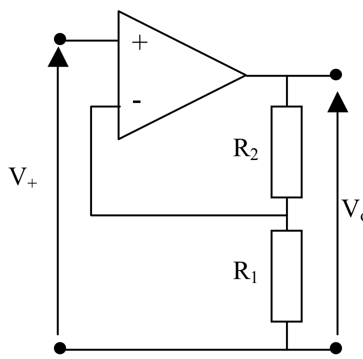
$$V_o=A(V_+-V_-)$$

Where the gain A can be as high as 10^5 to 10^6. In this raw form the circuit isn't actually much use, the gain is often not that well specified and in any case is so high that for many applications it would be over the top. What happens in practice is we build other components around the op-amp that allows control of the gain by a technique called negative feedback.

Negative feedback

Feedback describes the situation where a fraction of the output signal is fed back into the input of a system. Both positive and negative feedback can be used where the fed back portion of the signal either adds or subtracts from that already there, and both have useful application. Positive feedback can lead to instability but also to stable oscillations and forms the basis of many oscillatory signal generator circuits. Negative feedback, on the other hand, leads to stable circuit operation and better control of several other circuit parameters.





The circuit above left shows a simple implementation of negative feedback. A circuit connected between the output of the op-amp and its inverting input (V_-) feeds a fraction α of the output back into the input. For instance the network of resistors shown in the circuit on the right acts as a potential divider feeding back a fraction $\alpha=R_1/(R_1+R_2)$ of the output voltage to the inverting input of the op-amp. Thus we can write:

$$V_- = \alpha V_o = \frac{R_1}{R_1 + R_2} V_o$$

But of course the output of the op-amp is also dependant on its inputs via its gain A, so:

$$V_o = A\left(V_+ - V_-\right) = A\left(V_+ - \alpha V_0\right)$$

Re-arranging this equation to find V_o we get:

$$V_o = \frac{AV_+}{1+\alpha A} = \frac{V_+}{\alpha + 1/A}$$

If we now remember that A is very large and we also choose α>>1/A then we can ignore the 1/A factor in the denominator and the equation simplifies to:

$$V_o = \frac{V_+}{\alpha} = \frac{R_1 + R_2}{R_1} V_+ = \left(1 + \frac{R_2}{R_1}\right) V_+$$

In other words we have created a circuit with a controlled gain $G=1/\alpha=(1+R_2/R_1)$ that is set simply by the resistor values R_2 and R_1.

In addition the negative feedback also gives us further advantages.

1. In general the op-amp will have non-zero output impedance. The negative feedback acts to reduce the apparent output impedance by a factor of $\sim 1/(\alpha A)$.
2. The op-amp will also have non-infinite input impedance. The negative feedback tends to increase this apparent input impedance by a factor $\sim\alpha A$.

Analysing op-amp circuits and the virtual earth approximation

So all round the negative feedback appears to be a good thing. However the analysis of op-amp circuits is rather cumbersome when done in terms of feedback factors etc.. Fortunately there are a number of entirely reasonable approximations that we can make regarding the operation of an op-amp in a circuit using negative feedback that are summed up as the ***virtual earth approximation***. These assumptions are:

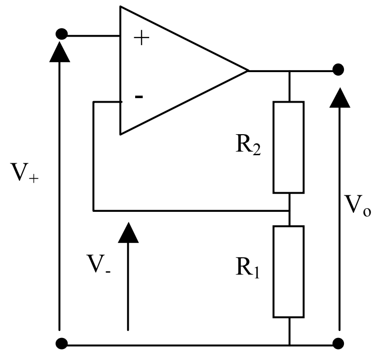
1. **The two inputs are at the same voltage: $V_+=V_-$.** This is best understood by observing that the output voltage must lie somewhere between the supply voltages ($-15V<V_o<+15V$ say) and that therefore if the raw op-amp gain A is very large (say 10^5) then the input voltage difference must be very small ($-0.15mV<V_+-V_-<+0.15mV$).
2. **The inputs draw no current**. In reality the raw input impedance of the op-amp can be very high ($10^{10}\Omega$ would not be unreasonable for some). Along with the increase in this due to the benefits of the negative feedback the assumption of no current draw is entirely reasonable.
3. **The output impedance of the op-amp is zero**. The raw output impedance of the op-amp is in any case low (a few Ω to a few $k\Omega$ in general). Once we have applied negative feedback the effective *circuit* output impedance drops even further and hence the assumption of zero output impedance becomes reasonable.

Based on these assumptions we can now think about analysing real circuits. You might notice that virtual earth approximation makes no mention of op-amp gain A. This is because the basis of the virtual earth approximation, that there has been the successful implementation of negative feedback, has ensured that the raw gain of the op-amp is now irrelevant!

We can now proceed to see how he virtual earth approximation lets us calculate the gain of some real circuits.

Non-inverting amplifier

Above we calculated the gain of a non-inverting amplifier (output voltage has the same sign as the input voltage) by considering negative feedback itself. The diagram below shows the circuit again:



Our input in this case is connected to V_+, so $V_+=V_i$. Applying the rules of the virtual earth approximation we can say that $V_-=V_+=V_i$ also. We can then apply Kirchhoff's current law at the node V_- noting from the other rules of the virtual earth approximation that the V_- terminal draws no current, and that the output of the op-amp has no output resistance and hence looks like a perfect voltage source of value V_o. Using these rules we can sum currents at V_- in the same manner as we do for node voltage analysis to get:

$$\frac{(V_o - V_-)}{R_2} + \frac{(0 - V_-)}{R_1} = 0$$

Or substituting $V_-=V_i$:

$$\frac{(V_o - V_i)}{R_2} + \frac{(0 - V_i)}{R_1} = 0$$

By a simple rearrangement of the above equation we get:

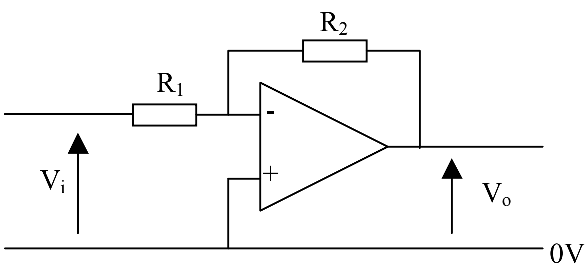
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i$$

That is, the same equation that we derived above using a negative feedback approach.

[As a special case of the non-inverting amplifier circuit when $R_2=0$ and $R_1=\infty$ (an open circuit) we get the "buffer amplifier" with a gain of only 1. Though not at first site very useful this type of circuit is very important as it can be added to the output of some preceding circuit which may have a significant output impedance. It won't load that circuit itself as it has an extremely high input impedance, and it also has an extremely low output impedance for driving following circuits.]

Inverting amplifier

The inverting amplifier is another simple yet useful op-amp circuit. It is called an inverting amplifier as it has a negative gain – the output voltage has the opposite sign to the input voltage. The circuit for an op-amp inverting amplifier is shown below:



As above we can now apply the rules of the virtual earth approximation. Firstly $V_-=V_+=0V$ (assuming all other voltages are measured relative to this line). Secondly, we assume that the inverting input takes no current and that the output of the op-amp has no output impedance and behaves like a perfect voltage source. We now use these assumptions when we sum currents at the inverting input as above in the same manner as for node-voltage analysis:

$$\frac{(V_o - V_-)}{R_2} + \frac{(V_i - V_-)}{R_1} = 0$$

Or substituting $V_-=0$:

$$\frac{(V_o - 0)}{R_2} + \frac{(V_i - 0)}{R_1} = 0$$

By a simple rearrangement of this equation we get:

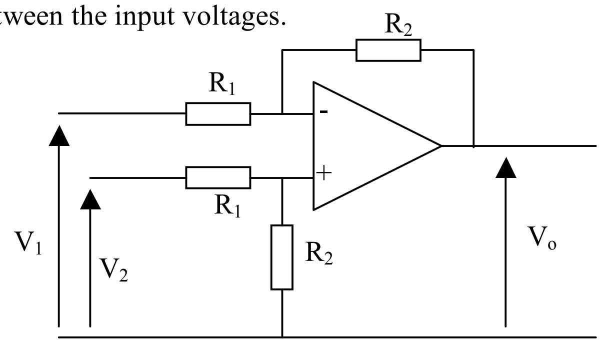
$$V_o = -\frac{R_2}{R_1} V_i$$

Thus the inverting amplifier has a gain of $-R_2/R_1$.

[A particular special case of the inverting amplifier is the trans-impedance amplifier where we set $R_1=0$. As can be easily seen from the above equation, this results in an infinite voltage gain!!? However, looking at the circuit in a slightly different way by connecting a current source I_i (such as a photodiode) at the input we can see that $V_o=-I_iR_2$. In other words the circuit converts current to voltage.]

More complex circuits

The same approach can be taken for even more complex circuits (including those with capacitors or inductors). As an example the diagram below shows a differential amplifier whose output voltage is the difference between the input voltages.



In this case we can still say that $V_+=V_-$ but we cannot definitively say exactly what this voltage is (as we could in the two previous cases). We assign this unknown voltage the symbol $V= V_+=V_-$ but as an unknown that we must eliminate it in order to get a relationship between V_o, V_1 and V_2. We therefore need two equations and we can get these from summing currents at both V_+ and V_-:

$$\text{at } V_+ : \quad \frac{(V_2 - V)}{R_1} + \frac{(0 - V)}{R_2} = 0$$

$$\text{at } V_- : \quad \frac{(V_1 - V)}{R_1} + \frac{(V_o - V)}{R_2} = 0$$

Solving these simultaneous equations by eliminating our unknown V yields the result:

$$V_o = \frac{R_2}{R_1}(V_2 - V_1)$$

[Compare this approach to that using superposition in the 2^{nd} classwork]