Norton and Thévenin equivalent circuits and the principle of superposition

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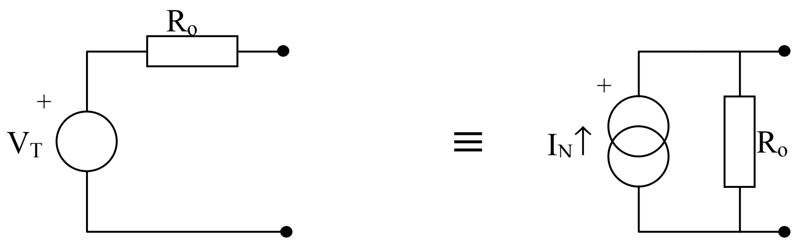
The circuits we use in real life are often very complex. The signal generators and oscilloscopes you use in first year laboratory for example contain several hundred components. When we connect such a device to another circuit or instrument it is very useful to be able to reduce these complex circuits to a greatly simplified but still accurate "equivalent circuit" to allow us to work out what will happen when we connect other circuits to them. In many cases we can treat our complex circuit as a "black box" that contains only two components, a voltage or current generator and a single resistor or impedance. This is formalised in the two theorems below.

Thévenin's Theorem

"Any two terminal network of resistors and EMF sources is equivalent to a single voltage source V_T in series with a single resistor R_o."

Norton's Theorem.

"Any two terminal network of resistors and EMF sources is equivalent to a single current source I_N in parallel with a single resistor R_o."



Thévenin equivalent circuit — Norton equivalent circuit

And of course if both these circuits are equivalent to any circuit with two terminals then they must also be equivalent to each other.

Voltage sources, current sources, short circuits and open circuits

Key to understanding these equivalent circuits is to understand the particular components (voltage and current sources) that Thévenin and Norton define in their equivalent circuits. It is important to realise that these are idealised circuit elements that have properties that are difficult to find in the real world. However, that aside, the idealised circuit elements have great use in simplifying our understanding of circuit operation in general.

In addition, two further concepts are very useful in understanding why ideal voltage and current sources could never exist, but also how we analyse circuits to determine their Thévenin and Norton equivalents. These two simple concepts are the short circuit and the open circuit.

A short circuit is simply a piece of perfectly conducting wire places between two nodes on a circuit. It is called a short circuit because it effectively bypasses any component it is placed in parallel with. This has the effect of reducing the current in those components to zero while the short circuit carries a current which we shall refer to later as the short circuit current I_{SC}. Of course the other aspect of the short circuit is that having no resistance the voltage across a short circuit must be identically zero ($V_{SC}\equiv0$).

An open circuit is even more trivial. It is no electrical connection between two nodes. There is no current flowing in an open circuit ($I_{OC}\equiv0$). But of course we can now measure a voltage across the open circuit, V_{OC}. Note that measuring the open circuit voltage between two nodes, does not imply that we remove all other components between those nodes, simply that we do not attach any extra components.

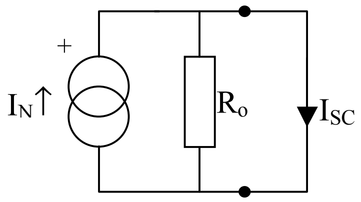
An ideal voltage source is a circuit element that maintains a fixed voltage across its terminals irrespective of what is connected between those terminals. This therefore implies that the ideal voltage source is capable of delivering an arbitrary current. This of course gives us problems should we wish to put a short circuit between the terminals. In these circumstances the voltage source would have to deliver an infinite current in order to maintain the appropriate terminal voltage and consequently would have to be able to deliver an infinite power. On the other hand voltage sources get on extremely well with open circuits.

An ideal current source is more of an unfamiliar concept because at least an ideal voltage source is close to what we understand a battery to be. By contrast there is no familiar equivalent to a current source. An ideal current source is a circuit element that delivers a fixed current irrespective of what is connected to its terminals. Here we now have the problem of what happens when we don't connect anything at all across its terminals, ie an open circuit. Under these circumstances the voltage on the terminals of the current source will rise and rise to infinity in an attempt to push current through the open circuit. Current sources like short circuits.

So there are problems with ideal current and voltage sources, but the Thévenin and Norton equivalent circuits also have those resistors in them. It is these resistors that make their behaviour acceptable to us in the real world – we can no longer short circuit the voltage source, nor open circuit the current source.

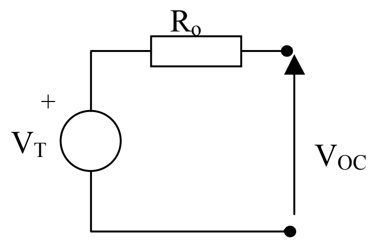
Deriving the Thévenin and Norton equivalent circuit parameters

The key to understanding how we can determine the equivalent circuit parameters is to recognise that as equivalent circuits, any experiment we perform on the equivalent circuit would produce exactly the same results if we were to perform it on the more complex circuit.



If we want to measure I_N then we should place a short circuit across the equivalent circuit terminals. When we do this no current flows down R_o and the whole of I_N flows down our short circuit. We can thus state that $I_N = I_{SC}$.

On the other hand if we want to measure V_T then all we need to do is measure the open circuit voltage across the equivalent circuit terminals. With the terminals open circuited, no current flows through R_o, there is no voltage drop across R_o and so we see the whole of V_T at the terminals. We can thus state that $V_T = V_{OC}$.



We can probably also see that if we want to measure R_o then we should replace the current source in the Norton equivalent by an open circuit or the voltage source in the Thévenin equivalent by a short circuit. Having done that all we have connected between the terminals is the resistor R_o., which we can then measure.

Alternatively we can see that by short circuiting the Thévenin equivalent we will draw a current $I_{SC} = I_N = V_T/R_o$. Or likewise by measuring the open circuit voltage across the Norton equivalent circuit we will see a voltage $V_{OC} = V_T = I_N R_o$. So we can also state that $R_o = V_{OC}/I_{SC} = V_T/I_N$.

Doing it to a real circuit

So now we know how to determine the circuit parameters for the equivalent circuits themselves, all we need to do is repeat these processes on the more complex circuit. Thus:

1) V_T is the open circuit voltage V_{OC} at the output of the circuit.
2) I_N is the current that would flow in a short circuit placed across the output of the circuit, I_{SC}.
3) R_o is the ratio of V_{OC}/I_{SC}

or alternatively…

4) R_o is the resistance between the terminals if we replace all the voltage sources with short circuits and all the current sources with open circuits.

Performing these "tests" analytically on a circuit may not be trivial and may involve using a variety of circuit analysis techniques such as superposition, Kirchhoff's laws, mesh or nodal analysis or just simple circuit manipulations. Performing these tests in real life on a circuit is almost certain to produce problems and is most certainly not to be recommended – first year lab wouldn't take nicely to you shorting out all their instrumentation!

So why bother?

There are two reasons that we are interested in Thévenin and Norton equivalent circuits. The first is that it enables us to determine what happens when we connect something else to our circuit. The classic case would be what you are plugging the output of your signal generator into in the lab. The receiving circuit will have an input impedance (a resistance between its receiving terminals) and we can therefore determine how much signal actually gets to the receiving circuit by considering the action of a simple potential divider.

The second reason for understanding Thévenin and Norton equivalent circuits is that recognising that the two are interchangeable becomes a useful tool in circuit simplification itself. Changing parts of a circuit between the two can result in combinations of circuit elements e.g. resistors in series or parallel or even combinations of sources that can be trivially combined into a single element.

The principle of superposition

The principle of superposition is inextricably linked with both Thévenin's original description of his equivalent circuit but also with many of the concepts we have already discussed with regards to both Norton and Thévenin equivalent circuits. In general physical terms it is a principle that finds application in all sorts of circumstances. In general anywhere where there is a linear relationship between system parameters, as in electrical circuits between current and voltage by Ohm's law, then the principle of superposition applies.

For electrical circuits it can be summed up by the following statement:

> In a circuit containing several voltage and/or current sources, any voltage or current in that circuit is a linear superposition of the current or voltage at that point in the circuit that would result from each source considered separately with the other voltage sources replace by short circuits and the current sources replaced by open circuits.

Application of the principle of superposition can be a simple but powerful technique in circuit analysis. Not only because it lets us instantly simplify the process of dealing with complex circuits with multiple sources, but also because it allows us to think of about different signal paths in a circuit.

For instance, to apply the principle of superposition in circuit containing several sources where we want to find the Thévenin equivalent, we simply work out the V_{OC} that would appear at the output when we leave each of the sources in the circuit alone in turn, while removing all the other sources.