Electronics Classwork 4 - Solutions, 27th January 2005

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1. Potential divider:

$$\frac{\tilde{V}_o}{\tilde{V}_i} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}}$$

(a)

$$\text{as } \omega \to 0 : \frac{\tilde{V}_o}{\tilde{V}_i} \to \frac{j\omega \frac{L}{R}}{1} = j\omega \frac{L}{R}$$

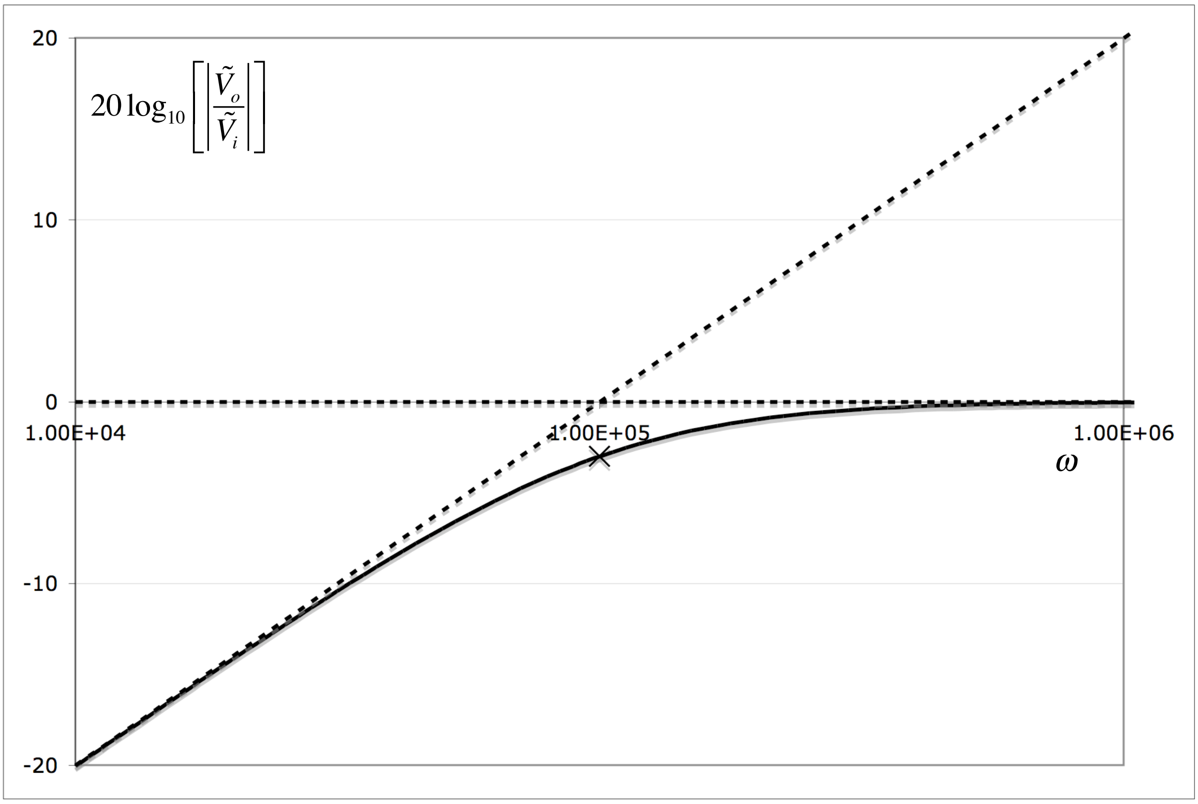
$$\text{as } \omega \to \infty : \frac{\tilde{V}_o}{\tilde{V}_i} \to \frac{j\omega \frac{L}{R}}{j\omega \frac{L}{R}} = 1$$

$\omega \to 0$ asymptote is straight line of +20dB/decade crossing 0dB at $\omega = R/L = 10^5$ rad s^{-1}

$\omega \to \infty$ asymptote is a straight horizontal line at 0dB

(b) $\omega_{3dB} = 10^5$(see above) $f_{3dB} = \omega_{3dB}/(2\pi) = 15.9$kHz

(c)



(d) High pass filter

2. (a)

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{1 + j\omega \frac{L}{R} - \omega^2 LC}$$

(b)

$$\tilde{V} = \tilde{I}_o Z = \tilde{I}_o \frac{j\omega L}{1 + j\omega \frac{L}{R} - \omega^2 LC}$$

(c)

$$\tilde{I}_R = \frac{\tilde{V}}{R} = \tilde{I}_o \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R} - \omega^2 LC} = \tilde{I}_o \frac{A\omega}{1 + j\frac{\omega}{\omega_o Q} - \frac{\omega^2}{\omega_o{}^2}}$$

$$A = j\frac{L}{R}; \omega_o = \frac{1}{\sqrt{LC}}; Q = \frac{R}{\omega_o L}$$

(d)

$$\omega_o = 2\pi f = 2\pi \times 125{,}000 = 7.85 \times 10^5 \text{ rad s}^{-1}$$

$$C = \frac{1}{\omega_o{}^2 L} = 463nF$$

$$Q = \frac{R}{\omega_o L} = 36.4$$

(e) At resonance $\omega = \omega_o$

$$\tilde{V} = \tilde{I}_o \frac{j\omega L}{1 + j\omega \frac{L}{R} - 1} = \tilde{I}_o \frac{j\omega L}{j\omega \frac{L}{R}} = \tilde{I}_o R = 0.1 \times 100 = 10V$$

$$\tilde{I}_R = \frac{\tilde{V}}{R} = 0.1A$$

$$\tilde{I}_C = \tilde{V} j\omega_o C = 3.64\, jA$$

$$\tilde{I}_L = \frac{\tilde{V}}{j\omega_o L} = -3.64\, jA$$

(f) If there were no capacitor the current source would have to supply both the 100mA for the resistor and the 3.64A for the inductor. However these would be 90° out of phase and so would add in quadrature to give a total current of $\sqrt{(0.1^2+3.64^2)}=3.64$A

3.

$$\frac{\tilde{I}_R}{\tilde{I}_o}=\frac{j\omega\frac{L}{R}}{1+j\omega\frac{L}{R}-\omega^2LC}$$

$$\text{as } \omega\to 0:\frac{\tilde{I}_R}{\tilde{I}_o}\to\frac{j\omega\frac{L}{R}}{1}=j\omega\frac{L}{R}$$

$$\text{as } \omega\to \infty:\frac{\tilde{I}_R}{\tilde{I}_o}\to\frac{j\omega\frac{L}{R}}{-\omega^2LC}=\frac{1}{j\omega RC}$$

$$\text{at } \omega=\frac{1}{\sqrt{LC}}:\frac{\tilde{I}_R}{\tilde{I}_o}\to\frac{j\omega\frac{L}{R}}{1+j\omega\frac{L}{R}-1}=1$$

$\omega\to 0$ asymptote is straight line of +20dB/decade crossing 0dB at $\omega=R/L=2.86\times10^7$ rad s^{-1}

$\omega\to\infty$ asymptote is straight line of -20dB/decade crossing 0dB at $\omega=1/RC=2.16\times10^5$ rad s^{-1}

$\omega=\omega_o= 7.85\times10^5$ graph must pass through 0dB