Electronics Classwork 3 - Solutions, 13th January 2005

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1. (a) 22mH and 47mH in parallel form a combined inductance of 22x47/(22+47)=15.0mH. In series combination with the 33mH inductor the total inductance is 15.0+33=48.0mH

(b) 20nF in series with 20nF gives combined capacitance of 20x20/(20+20)=10nF. 10nF in parallel with 10nF gives 10+10=20nF.

2. Kirchhoff's voltage law on loop

$V=IR+V_C$

But from Ohms law on capacitor:

$I=CdV_C/dt$

So

$V=RC\ dV_C/dt + Vc$

Or

$dV_C/dt=(V-V_C)/RC$

Integrate:

$$\int_0^{V_C} \frac{dV_C}{V_C - V} = \int_0^t \frac{-dt}{RC}$$

Limits given by $V_C=0$ at t=0 and $V_C=V_C$ at time t=t.

$$\left[\ln\left(V_C - V\right)\right]_0^{V_C} = \left[\frac{-t}{RC}\right]_0^t$$

$$\ln\left(V_C - V\right) - \ln(-V) = \ln\left(\frac{V - V_C}{V}\right) = \frac{-t}{RC}$$

Lake exponential of both sides:

$$\frac{V - V_C}{V} = \exp\left(\frac{-t}{RC}\right)$$

$$V_C = V\left(1 - \exp\left(\frac{-t}{RC}\right)\right)$$

$V=V_o=10V$, $\tau=RC=10^3 x10^{-6}=10^{-3}s=1ms$

Voltage across resistor $V_R=V-V_C$; $I=V_R/R$

$$V_C = V\left(\exp\left(\frac{-t}{RC}\right)\right); \qquad I = \frac{V}{R}\left(\exp\left(\frac{-t}{RC}\right)\right)$$

3. Take real part of complex value:

(a) $I=I_o\cos(\omega t)$

Amplitude I_o, angular frequency=ω, frequency=$\omega/(2\pi)$, phase=0

(b) $V=240\sqrt{2}\cos(100\pi t+\pi/2)=-339.4\sin(100\pi t)V$

Amplitude 339.4V, f=50Hz, $\omega=2\pi x50=100\pi$, phase=$\pi/2$ radians=90°

Convert to cos then change cos term to exp(j…) and drop the exp[jωt] term

(c) $\tilde{V}=5.0\exp[j\pi/4]$

Amplitude 5.0V, ω=1000, f=1000/(2π)=159Hz, phase=$\pi/4$ radians=45°

(d) $I=7.0\sin(62.8x10^6 t - \pi/6)= 7.0\cos(62.8x10^6 t -2\pi/3)$

$\tilde{I}=7.0\exp[j(62.8x10^6 t - 2\pi/3)]$

Amplitude 7.0V, $\omega=62.8x10^6$, $f=62.8x10^6/(2\pi)=10^7$ Hz=10MHz, phase=$-2\pi/3$ radians=-120°

4. Complex impedance of capacitor, $Z_C=1/(j\omega C)$; Complex Ohm's Law $V=IZ_C$

(b) $Z_C=1/(j\omega C)=-j/(100\pi\ 10^{-9})= -3.18x10^6j$

$\tilde{I}=\tilde{V}/Z_C= 240\sqrt{2}\exp[j\pi/2]/ -3.18x10^6j=1.07x10^{-3}j\exp[j\pi/2]$

$= 1.07x10^{-3}\exp[j\pi]=-1.07x10^{-3}$

Real current, take real part:

$I=-1.07x10^{-3}\cos(100\pi t)A=-1.07\cos(100\pi t)$ mA

(c) $Z_C=1/(j\omega C)=-j/(1000x10^{-9})= -10^6j$

$\tilde{I}=\tilde{V}/Z_C= 5\exp[j\pi/4]/ (-10^6j)=5x10^{-6}j\exp[j\pi/4]$

$= 5x10^{-6}\exp[j3\pi/4]$

Real current, take real part:

$I=5x10^{-6}\cos(1000\pi t+3\pi/4)A=5\cos(1000\pi t+3\pi/4)\ \mu A$

5. Complex impedance of inductor, $Z_L=j\omega L$; Complex Ohm's Law $V=IZ_L$

(a) $Z_L=j\omega L=j10^{-6}\omega$

$\tilde{V}=\tilde{I}Z_L= I_o\ j10^{-6}\omega\exp[jo]= I_o\ 10^{-6}\omega\exp[j\pi/2]$

Real current, take real part:

$V= I_o\ 10^{-6}\omega\cos(\omega t+\pi/2)=- I_o\ 10^{-6}\omega\sin(\omega t+\pi/2)$

(b) $Z_L=j\omega L=j62.8x10^6x10^{-6}=62.8j\Omega$

$\tilde{V}=\tilde{I}Z_L= 7x62.8j\exp[-j\pi/6]= 439.6\exp[j\pi/3]$

Real voltage, take real part:

$V=439.6\ Re[\exp[j(\omega t+\pi/3)]] =439.6\cos(62.8x10^6 t+\pi/3)$