Electronics Classwork 1 - Solutions, 6th January 2005

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1. Resistance in copper

(a) Key equation: $R=\rho_e L/A$,

[R= resistance, L=length of wire, A= Cross sectional area]

Rearrange to give: $L=RA/\rho=1 \times 0.25\times10^{-6}/(1.7\times10^{-8}) = 14.7$m

(b) Key equation: $1/\rho_e=\sigma=\mu ne$

[σ=conductivity, μ=mobility, n=electron density, e=electron charge]

Need to find n:

Number of moles of copper per m^3 , $N_{Cu}= \rho_{cu}/W_{cu}$

Number of copper atoms per m^3, $n_{Cu}=N_{Cu}xN_A=N_A\rho_{cu}/W_{cu}$

2 free electrons per Cu atom so number of electrons per m^3, $n=2xn_{Cu}=2N_A\rho_{cu}/W_{cu}$

Rearrange conductivity equation in terms of mobility μ:

$\mu=1/(\rho_e ne)=W_{Cu}/(2N_A\rho_{Cu}\rho_e e)=63.5/(2 \times 6.02\times10^{23} \times 8{,}920\times10^3 \times 1.7\times10^{-8} \times 1.6\times10^{-19})$

$=0.0022m^2$

(c) Key equation: $v_d=\mu_e E$, $E=V/L$

Calculate electric field along wire from $E=V/L=1/14.7=0.068\ Vm^{-1}$

Drift velocity given by $v_d=\mu_e E=0.0022*0.068=0.00015ms^{-1}=0.15mms^{-1}$

(d) Key equation: $eV=1/2\ mv^2$

Electric energy is converted into kinetic energy, rearrange equation in terms of v to give

$V=\sqrt{(2eV/m_e)}= \sqrt{(2 \times 1.6\times10^{-19} \times 1/(9.1\times10^{-31}))}=590{,}000ms^{-1}$

2. Resistance in silicon

(a) Key equation: $\sigma=\sigma_e+\sigma_h=\mu_e n_e e+\mu_h n_h e=n_i e(\mu_e+\mu_h)$

Put in values: $\sigma=n_i e(\mu_e+\mu_h)=1.45\times10^{16} \times 1.6\times10^{-19} \times (0.14+0.045) = 4.3\times10^{-4}\Omega^{-1}m^{-1}$

Resistivity $\rho=1/\sigma=2{,}300\Omega m$

11 orders of magnitude higher than copper!!

(b) Key equation: $\sigma=\sigma_e+\sigma_h=\mu_e n_e e+\mu_h n_h e$

Number of electrons now very much higher than number of holes so

$\sigma=\sigma_e=\mu_e n_e e=0.14 \times 10^{22} \times 1.6\times10^{-19} = 224\Omega^{-1}m^{-1}$

(c) Key equation: $R=\rho L/A$

Tolerance on resistance is 5% so try setting width of strip accurate to 5% ie $20\mu m \pm 1\mu m$

$L=RA/\rho=RA\sigma=10^4 \times 10^{-6} \times 20\times10^{-6} \times 224 = 44.8\mu m$

Total accuracy of resistance will depend on length too – are the tolerances correlated? Could double width and length to get better accuracy

3. Star/Delta transformation

(a) Follow the hint:

Resistance between A and B when C is disconnected:

for star = R_1 in series with R_2 = R_1+R_2

for delta = R_x in parallel with (R_y in series with R_z) = R_x // $(R_y+R_z)=R_x(R_y+R_z)/(R_x+R_y+R_z)$

so:

$$R_1+R_2 = R_x(R_y+R_z)/(R_x+R_y+R_z) \qquad (1)$$

And similarly by permutation:

$$R_2+R_3 = R_y(R_z+R_x)/(R_x+R_y+R_z) \qquad (2)$$

$$R_3+R_1 = R_z(R_x+R_y)/(R_x+R_y+R_z) \qquad (3)$$

Three simultaneous equations, solve for R1 by calculating (1)+(3)-(2) to give:

$$2\ R_1 = 2\ (R_xR_y+R_xR_z+R_zR_x+R_zR_y-R_yR_z-R_yR_x)/(R_x+R_y+R_z)=2R_xR_z/(R_x+R_y+R_z)$$

$$\text{or } R_1= R_xR_z/(R_x+R_y+R_z)$$

Other 2 equations can be found by permutation

(b) By symmetry $R_x+R_y+R_z=R$ and so:

$$R_1= R_xR_z/(R_x+R_y+R_z)= RR/(R+R+R)=R/3=1$$

So $R=3\Omega$

4. Resistor cube - simplest first!

R_{AC}:

A cube possesses three-fold symmetry. Assume a current I flowing into node A. This splits into 3 so I/3 flows in each resistor attached to node A. At the next node (say node B) the current further splits into 2 again by symmetry so I/6 flows in each of these 2 resistors. At the end of those resistors two currents of I/6 join again into a resistor flowing to node C. So all resistors connected to node C carry current I/3 too.

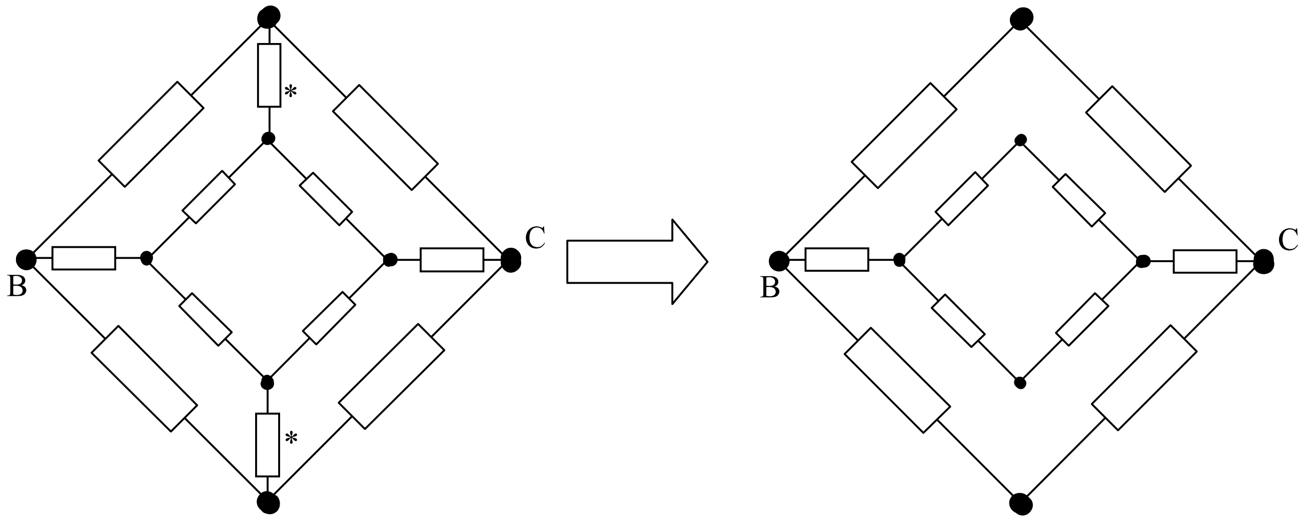
We can now work out the voltage from A to C by adding up the voltage drops along three resistors in a path from A to C:

$$V_{AC}=IR/3+IR/6+IR/3=IR(1/3+1/6+1/3)=IR\ 5/6=IR_{AC}$$

So $R_{AC}=5R/6=833\Omega$

R_{BC}:

Redraw circuit to look directly at face containing node B and C



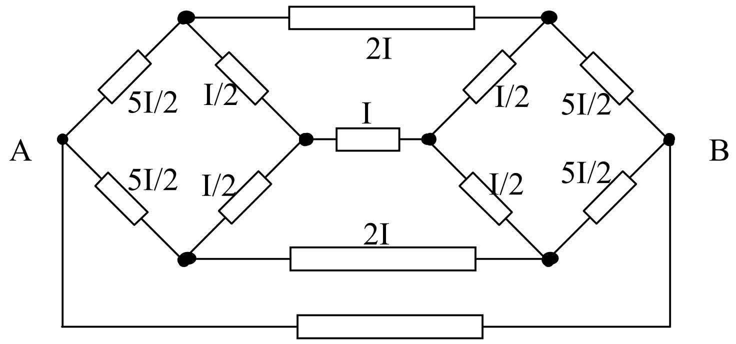
Resistors marked * carry no current as the voltage at either end must be equal so we can ignore them. The remaining resistors form two squares, an outer one directly connected to B and C and an inner one connected by a single resistor at each end to node B and C. The combined resistance of a square of 4 resistors of size R from corner to corner is simply R. We therefore have two parallel paths between B and C: The first of resistance R formed by the outer square, the second of resistance 3R formed by the inner square in series with the 2 connecting resistances. The total resistance is therefore:

R_{BC}=3R x R /(3R+R)=3R/4=750Ω

[This can also be done with star/delta transformations]

R_{AB}:

This is probably the most complicated but redraw the circuit to look directly at edge AB but move the resistor along AB out of the way to reveal the rest of the circuit.



Start by assuming a current of I in the central most resistor. This must split in 2 at either end by symmetry. The total voltage drop across all three resistors is 2IR so the single resistor in parallel with this combination must carry a current 2I. The result is that each resistor attached form here to nodes A and B must carry a current 5I/2. The total voltage drop from A to B is therefore 7IR but the current drawn is only 5I thus giving a resistance of 7R/5. Combining this in parallel with the lone resistor connected directly from A to B gives:

R_{AB}=R//(7R/5)=Rx7R/5/(R+7R/5)=7R/12=583Ω