Electronics Classwork 1, 6th January 2005

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You should attempt at least problems 1 and 3. Questions 2 and 4 (marked *) are more challenging but as a result are probably more interesting!

1. You are given a reel of copper wire of 0.25mm^2 cross-section and need to make a 1Ω resistor. Copper has a resistivity of $\rho_e=1.7 \times 10^{-8} \Omega \text{m}$.

(a) Using the resistor equation $R=\rho_e L/A$, calculate the length of wire will you need to achieve the desired 1 Ω resistance.

(b) You attach a 1V voltage source across the resistance to produce a 1A current flowing through it. From the conductivity equation $\sigma=\mu_e$ ne, make an estimate of the mobility, μ_e , of electrons in copper. [you will need to calculate n from the physical constants for copper given below]

(c) Calculate the electric field within your wire and hence the drift velocity, $v_d=\mu_e E$, of the electrons.

(d) Compare your answer to the velocity that would be achieved by directly accelerating the electron through free space with the same 1V source.

[Density of copper ρ_{Cu} =8,920 kg m⁻³, atomic mass of copper=63.5 g mol⁻¹, valency ("free" electrons per atom) of copper≈2, charge on an electron e=1.6x10⁻¹⁹ C, Avogadro's constant N_A=6.02x10²³ mol⁻¹, mass of an electron m_e=9.1x10⁻³¹kg].

2*. Semiconductor materials such as Silicon can have either negatively charged electrons and/or positively charged "holes" as carriers of charge in any current flow. In the particular case of intrinsic pure silicon the number of holes exactly equals the number of electrons $(n_e=n_h=n_i=1.45 \times 10^{16} \text{ m}^{-3} \text{ at room temperature})$. However the mobility of the holes $\mu_h=0.045 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$ is somewhat lower than that of electrons at $\mu_e=0.14 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$.

(a) The electron and hole currents effectively happen in parallel the electron and hole conductivities add and give $\sigma = \sigma_e + \sigma_h = \mu_e n_e e + \mu_h n_h e = n_i e(\mu_e + \mu_h)$. What is the combined resistivity of intrinsic silicon – compare this to copper.

(b) It is very difficult to achieve pure silicon and in fact most useful effects in silicon are obtained by doping with other elements that add extra electrons or holes. Calculate the resistivity of n-type doped silicon with an electron carrier density of 10^{22} m⁻³.

(b) You have to design a $10k\Omega$ 5% tolerance resistor that will be integrated onto a silicon chip. The resistor is made by diffusing an n-type dopant into a strip on the silicon chip to produce an electron carrier density of 10^{22} m^{-3} . Assume that your dopant diffuses uniformly to a depth of 1µm and that you are limited to producing features with a lateral resolution (accuracy) of 1µm. Produce a sensible design for your resistor that meets these design criteria

3. You will already be aware that combinations of resistors in series or parallel between two terminals can be modelled as a single resistance. When there are three terminals (A, B and C) to a network there are two simple ways of representing a simplified form of the circuit as shown in figure 1(a) the star formation and figure 1(b) the delta formation.



Figure 1 (a) Star formation of resistors



(a) Find an expression for the star resistances R_1 , R_2 and R_3 in terms of the delta resistances R_x , R_y and R_z that will ensure that the two networks are identical. [hint: try making sure that the resistances between pairs of terminals are identical when the third terminal is disconnected].

(b) If R_1 , R_2 and R_3 are all 1Ω what are the values of R_a , R_b and R_c in the equivalent delta circuit.

4*. Figure 2 shows a cube made up of identical $1k\Omega$ resistors. Your task is to find the resistance between one pair of the labelled nodes R_{AB} , R_{BC} or R_{AC} .



Figure 2: A cube of 1k resistors

For whichever case you choose, look carefully to try and think how you can simplify the problem and look particularly for symmetry. Are there any resistors that will always carry the same current as each other, or others that will carry no current at all because the voltage at each end of them is the same because of symmetry? Also do not be afraid to draw out the cube in a different distorted form, if that makes the symmetry more apparent. You might find that circuit transformations such as series/parallel conversions or even star/delta transformations are useful, but perhaps the most useful approach is to think about what happens if you assume an arbitrary current I or voltage V into part of the circuit and then look at how other currents and voltages in the circuit relate to that.