First Year Electricity and Magnetism Classwork 6

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3 March 2008

In all questions, choose the *best* answer.
Numerical answers need only be accurate to at most 2 significant figures.
You will need to draw some pictures and do some calculations. There are only 8 questions.

Good luck!

1. Magnetic field lines never end.

(A) True
(B) False

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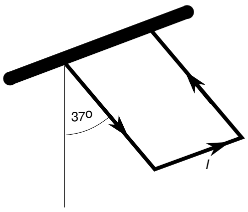
The correct answer is (A). There is no magnetic "monopole." So the flux through a closed surface must be zero, i.e., every field line that enters must exit somewhere else.

2. A flag has a wire around its edge, so that it can be "levitated." The flag is suspended from a horizontal pole; a current *I* is driven around the wire edge. In which direction is the magnetic field if the flag is levitated to a stationary position as shown in the diagram?

(A) Vertically upwards
(B) Vertically downwards
(C) To the left
(D) To the right
(E) More than one of the above



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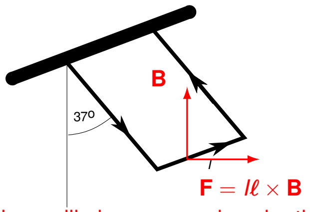
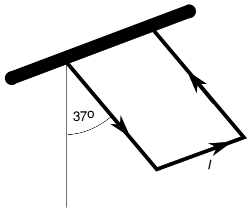


The correct answer is (A). The side legs will always cancel each other, so we only need to consider the horizontal edge. $\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$ needs to have a horizontal component, which requires $\mathbf{B}$ vertically. While a horizontal $\mathbf{B}$ could supply a vertical force, it will not provide the horizontal one needed to keep the flag in the orientation shown.

3. The flag in the previous question is square with sides of length 1 m. and weighs 16 N. If the magnetic field is 0.6 T, the current required to hold the flag at an angle of 37° as shown is

(A) 5.0 A
(B) 10.0 A
(C) 20.1 A
(D) None of the above



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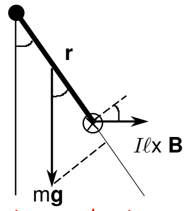

The correct answer is (B). Need to match the torque due to gravity by that due to I. Torque $\tau \equiv \mathbf{r} \times \mathbf{F}$. Note $m\mathbf{g}$ acts at centre of flag, hence $\mathbf{r} = \ell/2$ so $\tau_g = \frac{\ell}{2} mg \sin 37^\circ = I\ell B \cos 37^\circ$ and solve for I. [Take torques about flagpole so don't need tension along flag.]

4. A circular wire loop lies initially in the $x - y$ plane and carries a current I that flows in the right hand sense around the z-axis. A magnetic field $\mathbf{B} = B_o(\sin\theta, 0, \cos\theta)$. The loop will

(A) undergo accelerated translational motion
(B) rotate about the x-axis without any net translation
(C) rotate about the y-axis without any net translation
(D) rotate and translate
(E) None of the above

HINT: DRAW a PICTURE!!!

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The correct answer is (C). There is no net force on the loop, so it won't suffer translation, but there is a torque $\tau = \mu_m \times \mathbf{B}$ which is in the $(+\hat{\mathbf{y}})$ direction.

5. The wire loop in the previous question has a radius 2.5 m, lies initially in the $x - y$ plane and carries a current $I = 3$ A that flows in the right hand sense around the z-axis. The magnetic field $\mathbf{B} = 0.15(\sin 30^\circ, 0, \cos 30^\circ)$ T. The initial torque has a magnitude

(A) 7.07 Nm
(B) 8.84 Nm
(C) 4.42 Nm
(D) None of the above

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The correct answer is (C). The torque $\tau = \mu_m \times \mathbf{B} \equiv \mu_m B \sin\theta(+\hat{\mathbf{y}})$ with $\mu_m = IA = I\pi r^2$. Thus $|\tau| = (I\pi\, 2.5^2)\, 0.15 \sin 30^\circ$ which evaluates to (C).

6. The loop in the preceding question starts from rest. Thereafter it will

(A) rotate about the y-axis by 30° until μ_m is aligned the **B** at which point it will stop

(B) rotate in the same sense for a complete rotation

(C) rotate 90° past alignment of μ_m and **B**, at which point the torque will be a maximum in the opposite direction and it will reverse its sense of rotation

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(D) None of the above

The correct answer is (D). While it is true that $\tau \to 0$ when μ_m and **B** are aligned, its rotational inertia will carry it past that point. By the symmetry of the problem, it will stop when it has gone the same distance past the equilibrium position, i.e., 30° beyond alignment. After this, it will reverse its direction.

7. In lecture we found that the field on the axis of a circular loop of radius a carrying a current I is $\mathbf{B} = \frac{\mu_o}{2}\frac{Ia^2}{(x^2+a^2)^{3/2}}\hat{\mathbf{x}}$. Recalling the way we applied the Law of Biot and Savart, the field due to only the upper $(+\hat{\mathbf{y}})$ semi-circle of the loop is

(A) exactly half the expression we found for the whole loop, by symmetry.

(B) there will also be a y component of **B**

(C) Both (A) and (B)

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(B) there will also be a y component of **B**

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The correct answer is (C). All the current elements making up the loop will contribute the same dB_x as in the full loop, so the $\hat{\mathbf{x}}$-component is halved. But the symmetry is broken and so there will also be a y-component.

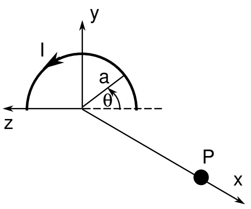
8. The field on axis due to only the upper $(+\hat{\mathbf{y}})$ semi-circle of a loop of radius a carrying a current I can be found by applying the Law of Biot and Savart $d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{I\, d\boldsymbol{\ell}\times\mathbf{r}}{r^3}$ with $d\boldsymbol{\ell} = a\, d\theta\, \hat{\boldsymbol{\theta}}$ with θ measuring the angle from the $-z$ axis, as shown, and

(A) $\mathbf{r} = (x, y, z)$

(B) $\mathbf{r} = x\hat{\mathbf{x}} - a\sin\theta\hat{\mathbf{y}} + a\cos\theta\hat{\mathbf{z}}$

(C) $\mathbf{r} = x\hat{\mathbf{x}} + a\sin\theta\hat{\mathbf{y}} + a\cos\theta\hat{\mathbf{z}}$

(D) None of the above



8. The field on axis due to only the upper $(+\hat{\mathbf{y}})$ semi-circle of a loop of radius a carrying a current I can be found by applying the Law of Biot and Savart $d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{I\, d\boldsymbol{\ell}\times\mathbf{r}}{r^3}$ with $d\boldsymbol{\ell} = a\, d\theta\, \hat{\boldsymbol{\theta}}$ with θ measuring the angle from the $-z$ axis, as shown, and

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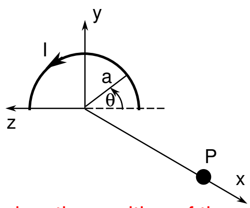

The correct answer is (B). **r** is defined as the position of the observation point relative to the current element, so $\mathbf{r} = (x, 0, 0) - (0, a\sin\theta, -a\cos\theta)$. As a bonus, you might like to find the $\hat{\mathbf{y}}$-component of the semi-circular loop by using these results and integrating, noting $\hat{\boldsymbol{\theta}} = \cos\theta\hat{\mathbf{y}} + \sin\theta\hat{\mathbf{z}}$.