First Year Electricity and Magnetism
Classwork 4
Quiz on Electricity

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19 February 2008

In all questions, choose the *best* answer.
Numerical answers need only be accurate to at most 2 significant figures.

Good luck!

1. A charge $+Q$ is placed at the origin. The resulting electric field is

$$E = \frac{Q}{4\pi\epsilon_o}\frac{1}{r^2}$$

(A) True
(B) False

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$$E = \frac{Q}{4\pi\epsilon_o}\frac{1}{r^2}$$

(A) True
(B) False

The correct answer is (B). The electric field is a *vector, so*

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_o}\frac{1}{r^2}\hat{\mathbf{r}}$$

2. Electrons are deposited on a conducting sphere of radius 0.1 m. The magnitude of the electric field at a distance of 1 m from the centre of the sphere is measured to be 9 N/C. Approximately how many electrons were deposited on the sphere?

(A) 10^{-9}
(B) 6.25×10^9
(C) 8.99×10^9
(D) None of the above

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The correct answer is (B). $E = q/4\pi\epsilon_o r^2$ so $9 = 8.99 \times 10^9 q/1^2$ $\Rightarrow q = 10^{-9}C = 10^{-9}C/1.6 \times 10^{-19}C/e$. (The radius of the conducting sphere is irrelevant as the distribution will be spherically symmetric.)

3. Two identical electric dipoles of strength p are placed next to one another separated along the x-axis by a distance D. If the vector dipole moments p are confined to the $y-z$ plane, what configuration corresponds to a stable equilibrium?

(A) Both dipoles will be aligned with one another
(B) The dipoles will be perpendicular to one another
(C) The dipoles will be anti-aligned with one another
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The correct answer is (C). The electric field of a dipole at points along the perpendicular to its dipole moment **p** is in the opposite direction to **p**. The stable configuration of a dipole in an electric field is with **p** aligned with **E** hence the dipoles will be anti-aligned with each other in a stable configuration.

4. A spherical distribution of charge has a charge density given by

$$\rho = \begin{cases} \rho_o \frac{r^2}{a^2} & r < a \\ 0 & r > a \end{cases}$$

The electric field for $r < a$ has magnitude

(A) $\frac{\rho_o}{\epsilon_o} \frac{r^3}{5a^2}$
(B) $\frac{\rho_o}{\epsilon_o} \frac{a^3}{5r^2}$
(C) $\frac{\rho_o}{\epsilon_o} \frac{r^3}{3a^2}$
(D) None of the above

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The correct answer is (A). Gauss's Law $\Rightarrow E 4\pi r^2 = \frac{1}{\epsilon_o} \int_0^r \rho_o \frac{r^2}{a^2} 4\pi r^2 dr$

so $E = \frac{\rho_o}{\epsilon_o 4\pi r^2} \frac{4\pi r^5}{5a^2}$

5. An irregularly shaped conductor with an internal cavity is supported by an insulating rod. A total charge $+Q$ is placed on the conductor. The charge

(A) Distributes itself around the inner surface bounding the cavity.
(B) Collects near the place where the insulating rod meets the conductor.
(C) Distributes itself around the outer surface in such a way that the electric field outside is the same as that due to a point charge $+Q$.
(D) Distributes itself throughout the body of the conductor so as to keep the electric field uniform there.
(E) None of the above.

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(B) Collects near the place where the insulating rod meets the conductor.
(C) Distributes itself around the outer surface in such a way that the electric field outside is the same as that due to a point charge $+Q$.
(D) Distributes itself throughout the body of the conductor so as to keep the electric field uniform there.
(E) None of the above.

The correct answer is (E). The charge will distribute itself over the *outside of the conductor;* ***E*** *is identically zero within the body of the conductor, but is not spherically symmetric outside.*

6. **Two conducting plates are joined along one edge to form a wedge shape with opening angle α. A charge $+Q$ is placed midway between the two plates, as shown in the sketch. The electric field between the two plates can be represented by that due to the charge $+Q$ together with:**

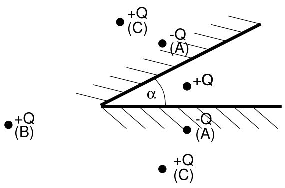
(A) Two images of charge $-Q$ at positions *A* as shown

(B) Two images of charge $-Q$ at positions *A* together with one of $+Q$ at position *B*.

(C) Four images as shown at *A* and *C*

(D) An infinite number of image charges.

(E) None of the above



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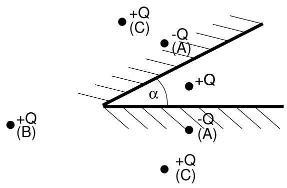

The correct answer is (D). *Q* will give the initial images $-Q$ at *A*, but each of these will generate an image $+Q$ to yield the configuration in *C*. But then these new image charges will also require further images.

7. A uniform electric field $\mathbf{E} = 5.0\hat{\mathbf{z}}$ N/C is established. You take a charge $Q = 2.0$ C and push it slowly from $z = 4$ m to $z = 2$ m. The amount of work that you do is

(A) -10.0 Nm

(B) +10.0 Nm

(C) -20.0 Nm

(D) +20.0 Nm

(E) None of the above

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(C) -20.0 Nm

(D) +20.0 Nm

(E) None of the above

The correct answer is (D). The work YOU do is $\int_{z=4}^{z=2} \mathbf{F}_{you} \cdot \mathbf{d}\ell$. If you move the charge slowly, $\mathbf{F}_{you} = -\mathbf{F}_E = -(2.0) \times 5.0\hat{\mathbf{z}} = -10.0\hat{\mathbf{z}}$. So the work you do is $-10.0\,[z]_4^2 = -10.0(2-4) = +20$ Nm. [A Nm is 1 Joule, of course.]

8. A sphere of radius r contains a charge Q distributed uniformly throughout its volume. Charge dQ is brought in from infinity and deposited uniformly over the surface. The potential energy of the charge dQ is

(A) $\frac{Q}{4\pi\epsilon_o r}$

(B) $\frac{Q\,dQ}{4\pi\epsilon_o r}$

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(D) None of the above

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(D) None of the above

The correct answer is (B). This follows naturally from the result for the potential of 2 point charges after noting (a) **E** due to *Q* is the same as a point charge and (b) smearing *dQ* around the sphere (i.e., tangential to the radial direction) doesn't change the energy.

9. If the charge density in the previous question is ρ_o, then $Q = \rho_o \frac{4}{3}\pi r^3$ while $dQ = \rho_o 4\pi r^2 dr$. Then the answer reduces to $dU = \rho_o^2 4\pi r^4 dr/3\epsilon_o$. Hence the total energy required to assemble a uniform sphere of radius R with total charge q is

(A) $\dfrac{3q^2}{20\pi\epsilon_o R}$

(B) $\dfrac{4q^2 R^5}{15\pi\epsilon_o}$

(C) $\dfrac{4q^2 R^5}{3\pi\epsilon_o}$

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(C) $\dfrac{4q^2 R^5}{3\pi\epsilon_o}$

(D) None of the above

The correct answer is (A), which follows by integrating dU from $r = 0$ to $r = R$ and setting $\rho_o = q/(4\pi R^3/3)$. So, if the electron is the limit of a uniformly charged sphere as it shrinks to a point, it's "self-energy" is infinite. If this disturbs you, you're in very good company.

10. An electric potential is given by $V(\mathbf{r}) = \alpha r$, where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ with $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$ and α is a constant. The electric field is

(A) $\alpha\hat{\mathbf{r}}$

(B) $\alpha\mathbf{r}$

(C) $\alpha\hat{\mathbf{r}}/2$

(D) $\alpha\mathbf{r}/2$

(E) None of the above

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(D) $\alpha\mathbf{r}/2$

(E) None of the above

The correct answer is (E). $\mathbf{E} \equiv -\nabla V = -\frac{\partial V}{\partial x}\hat{\mathbf{x}} + \ldots$. Now, for example, $\frac{\partial r}{\partial x} = \frac{1}{2}(x^2 + y^2 + z^2)^{-1/2} 2x = x/r$, so
$\mathbf{E} = -\alpha\left[(x/r)\hat{\mathbf{x}} + (y/r)\hat{\mathbf{y}} + (z/r)\hat{\mathbf{z}}\right] = -\alpha\hat{\mathbf{r}}$, which is *minus* Answer (A). Be careful with the minus sign.

11. A parallel plate capacitor has a net charge of 20 electrons on one plate that were removed from the other. The measured voltage across the plates is 5 Volts. The value of the capacitance is

(A) 3.2×10^{-18} F

(B) 6.4×10^{-19} F

(C) 1.6×10^{18} F

(D) There is not enough information given to determine the capacitance

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The correct answer is (B). Capacitance is the "capacity" to store charge, i.e., $C = Q/V = 20 \times 1.6 \times 10^{-19}/5$ F

12. Four identical capacitors are used in a circuit to store charge. There is a single, fixed voltage supply, V. In order to maximise the amount of charge stored, the capacitors should be connected:

(A) in parallel
(B) in series
(C) by putting pairs of capacitors in series, and connecting the two sets of pairs in parallel
(D) None of the above

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(C) by putting pairs of capacitors in series, and connecting the two sets of pairs in parallel
(D) None of the above

The correct answer is (A). The charge stored in the ith capacitor is $Q_i = CV_i$ so the obvious way to maximise the total charge is to maximise the voltage V_i across each capacitor. This is achieved by connecting them all in parallel, so the full voltage V appears across each of them.

13. A parallel plate capacitor with a vacuum between the plates is charged to a potential V_o. You then insert a dielectric material of dielectric constant ϵ_r into the region between the two plates. Given that the energy stored in a capacitor is $U = CV^2/2$, you

(A) had to do work in pushing the dielectric into the capacitor
(B) had to prevent the dielectric from being pulled in, i.e., it did work on you
(C) had to do work if the capacitor was left connected to the voltage source, otherwise it did work on you
(D) had to do work if the capacitor was disconnected from the voltage source, otherwise it did work on you

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The correct answer is (C). In all cases $C = Q/V = \epsilon_r Q_o/V_o$, i.e., C has increased. If you disconnected the capacitor, the charge Q_o says fixed, but the voltage is decreased by ϵ_r, so $U = U_o/\epsilon_r$. Less energy means you gained some. If you leave it connected, $V = V_o$ but C has increased. Thus $U = \epsilon_r U_o$ so the stored energy has increased.