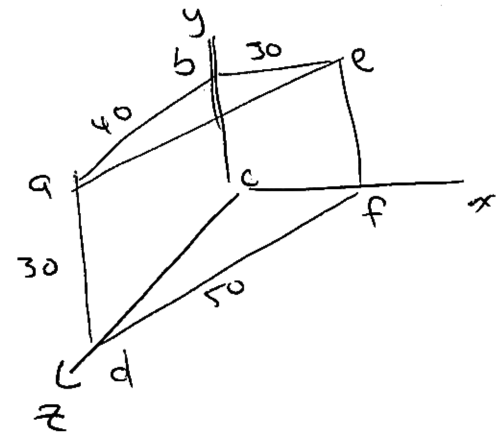
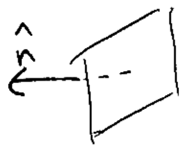
Solutions EM PS6 2008



(1) $\underset{\sim}{B} = 0.128\,\hat{z}$

a) abcd has outward normal $\hat{n} = -\hat{x}$



so $\Phi_B = \iint \underset{\sim}{B}\cdot\hat{n}\,dA = 0$

as $\hat{z}\cdot\hat{n} = 0$

b) befc has $\hat{n} = -\hat{z}$ (− sign for outward, but Q is a bit vague)

so $\Phi_B = \iint \underset{\sim}{B}\cdot\hat{n}\,dA = \iint (0.128\hat{z})\cdot(-\hat{z})\,dA$

$= -0.128\iint dA = -0.128 \times \underbrace{0.3\times 0.3}_{A} = \underline{\underline{-0.0115\ \mathrm{Tm}^2}}$

c) aefd has $\hat{n} = \frac{4}{5}\hat{x} + \frac{3}{5}\hat{z}$ (requires some trig!)

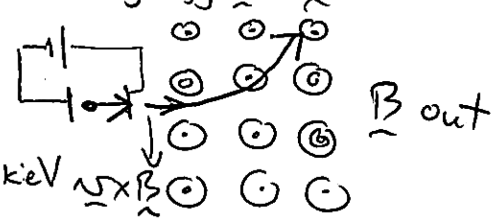
so $\Phi_B = \iint \underset{\sim}{B}\cdot\hat{n}\,dA = \iint 0.128\,\hat{z}\cdot\left(\frac{4}{5}\hat{x} + \frac{3}{5}\hat{z}\right)dA$

$= 0.128\times\frac{3}{5}A = 0.128\times\frac{3}{5}(0.5)(0.3) = \underline{\underline{+0.0115\ \mathrm{Tm}^2}}$

d) $\Phi_B(\text{net}) = \oiint \underset{\sim}{B}\cdot d\underset{\sim}{A} = 0$ by magnetic Gauss's Law.

Can also see that all other faces have $\underset{\sim}{B}\cdot\hat{n} = 0$ so answers to (b) & (c) must add to zero. Indeed, it would be easier to answer (c) by just using $\oiint \underset{\sim}{B}\cdot d\underset{\sim}{A} = 0$

(2) $R = \dfrac{mv}{|q|B}$ so $R^2 = \dfrac{2m(\frac{1}{2}mv^2)}{q^2B^2}$



Solve for $B^2 = \dfrac{2m(\frac{1}{2}mv^2)}{R^2e^2}$ Note: $\frac{1}{2}mv^2 = 2\,\mathrm{keV}$

Numbers: $B^2 = \dfrac{2(9.11\times10^{-31})\,2000\,\cancel{1.6\times10^{-19}}}{(0.18)^2(1.6\times10^{-19})^{\cancel{2}}} = 7.029\times10^{-7}$

so $B = \underline{8.38\times10^{-4}\ \mathrm{T}}$

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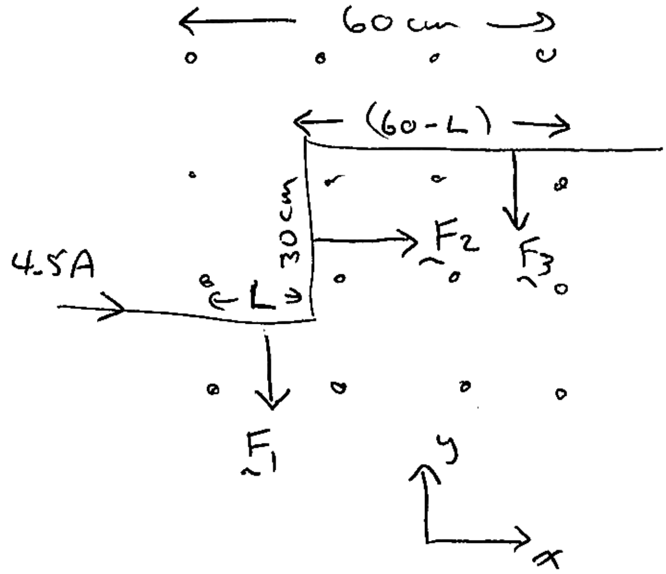


(3) $\underset{\sim}{F} = I \underset{\sim}{l} \times \underset{\sim}{B}$

split into 3 pieces and find $\underset{\sim}{F_1}$, $\underset{\sim}{F_2}$, $\underset{\sim}{F_3}$ as shown.

Set up x, y as sketched.

Then $\underset{\sim}{F_1} = ILB(-\hat{y})$

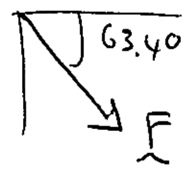
$\underset{\sim}{F_2} = I(0.3)B\hat{x}$

$\underset{\sim}{F_3} = I(.60 - L)B(-\hat{y})$

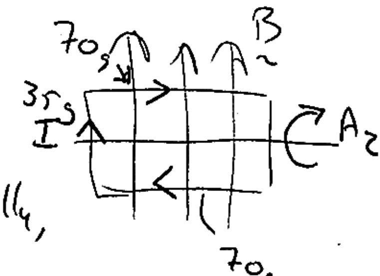
So $\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} = I(0.6)(-\hat{y})B + I(0.3)B\hat{x}$

$= 4.5(0.24)(0.3)\left[\hat{x} - 2\hat{y}\right] = 0.724\left(\frac{\hat{x}}{\sqrt{5}} - \frac{2\hat{y}}{\sqrt{5}}\right)$

$= 0.724$ at $63.4°$ below horizontal



(4) a) When I starts, can see ends have $\underset{\sim}{l} \parallel \underset{\sim}{B}$ so no force. Top & bottom give torque about A_z axis as shown



b) Quantitatively $\underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B} = I\underset{\sim}{A} \times \underset{\sim}{B}$

For I as shown $\underset{\sim}{A}$ (RH rule) into page & ⊥r $\underset{\sim}{B}$ initially,

so $|\tau| = IAB = 2 \times (1.00 \times 0.5)\, 3 = 3\ \text{Nm}$

Now $\tau = I_{inertia}\alpha$ so need moment of inertia about A_z.

$$I_{inert.} = \underbrace{70g \times (0.25)^2}_{top} + \underbrace{70g \times (0.25)^2}_{bottom} + \underbrace{2 \times 35g \times (0.5)^2/12}_{ends} = \frac{0.49}{48}$$

Thus $$\alpha = \frac{\tau}{I_{inert.}} = \frac{3}{0.49/48} = \underline{294\ \text{rad}/\text{s}^2}$$