Electricity & Magnetism, Problem Sheet 3 (Year 1)

1) Use the Drude model to estimate the conductivity of copper if the density of free electrons in copper is $8x10^{28}$ m^{-3}, and their mean time between collisions is $5x10^{-14}$ sec. Estimate the resistance of a copper wire of length 10km and radius 1mm.

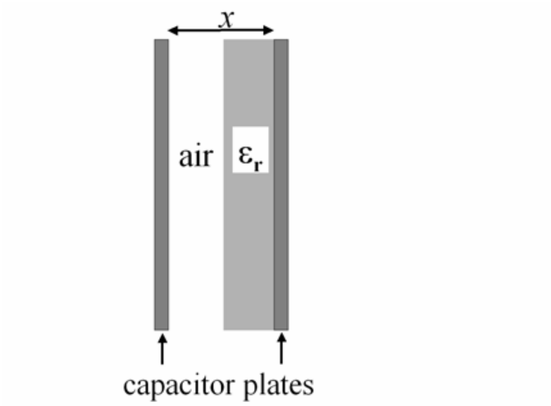
Mass of electron = $9.1x10^{-31}$ kg

Charge on an electron = $-1.6x10^{-19}$ Coulomb

2) An infinite slab of dielectric material with relative permittivity equal to 5 (ε_r=5) is placed in an electric field of 500Vm^{-1} with the field normal to the slab's surface. What is (i) the electric displacement (D) outside the slab (ii) the electric displacement inside the slab (iii) the electric field inside the slab (iv) the polarisation (P) inside the slab (v) the total charge (sum of free and polarisation charge) per unit area on each surface of the slab (Hint: use some form of Gauss's flux law).

$\varepsilon_0 = 8.854x10^{-12}$ F m^{-1}

3)



As shown in the diagram, half of the space between the plates of a parallel plate capacitor is filled with a medium with relative permittivity ε_r. The relative permittivity of air is 1. The area of each capacitor plate is *A*. Find an expression for the potential difference between the plates for a given charge density σ per unit area on the plates (Recall that the change in D across any interface is equal to the free charge density per unit area at the interface). Derive an expression for the capacitance of this arrangement.

For a practical capacitor, why is it important to make the dielectric medium fill the complete space between the capacitor plates and exclude an air gap?

4) Calculate the voltage across the wheels of a train at 200 km/hr from London to Bristol. The vertical component of the earth's magnetic field in London is about $4x10^{-4}$ Tesla, and the distance between the rails is 1.4 metre.

5) A cylindrical coil (solenoid) of length 100mm and radius 20mm has 100 turns. What are the values of *B* and *H* inside the coil if the coil is filled with air and the coil carries a current of 0.1 Amp? What are the values of *B* and *H* if the coil's interior is filled with a core made of a material with a relative permeability $\mu_r = 10^4$? What is the inductance in each case?

Permeability of free space = $4\pi \times 10^{-7}$ Hm^{-1}

6) (i) A circular coil with N turns and radius R rotates in a magnetic field B with angular frequency ω (period of rotation $=2\pi/\omega$). The rotation axis is perpendicular to the magnetic field. Use Faraday's law to show that the e.m.f. generated by the coil is $V = \omega\pi R^2 BN \cos(\omega t+\phi)$, where ϕ is a constant determined by the initial conditions.

(ii) It is proposed that a satellite might generate electrical power by rotating a coil with a 1km radius in the solar wind. The magnetic field in the solar wind is typically 10^{-9} Tesla. Suppose the coil has 10^4 turns, what rotation period would be needed to generate an alternating e.m.f. with a peak value of 1 Volt?

(iii) If it were possible to generate a reasonable e.m.f. in this way, what would be the source of the energy?

(iv) A second person suggests that it would not be necessary to rotate a coil. The solar wind moves at a velocity v = 400 km sec^{-1}, so a wire of length l (instead of a coil) stretched out in the solar wind will generate an e.m.f. of Bvl. What is the difficulty with this suggestion?

(v) (one to ponder at your leisure, not a serious question) A third person points out that the solar wind, being a plasma, is highly conducting, so it would be possible to complete a circuit which passes along a straight wire in one direction and returns through the solar wind. Would this stand a better chance of producing electrical power? Think of the train and the resistor as discussed in lectures.