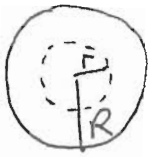
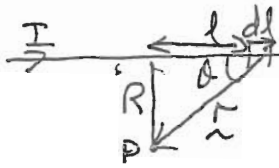
Yr 1 Electricity + Magnetism, Problem Sheet 2 (2005)

1) 

Current is spread uniformly, so $\pi R^2 j = I_0$ where j is the current density $I(r) = \pi r^2 j = \dfrac{I_0 r^2}{R^2}$

Ampere's law $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I \Rightarrow 2\pi r |\underset{\sim}{B}| = \dfrac{\mu_0 I_0 r^2}{R^2} \Rightarrow |\underset{\sim}{B}| = \dfrac{\mu_0 I_0 r}{2\pi R^2}$ for $r<R$

For $r>R$ $\quad 2\pi r |\underset{\sim}{B}| = \mu_0 I_0 \Rightarrow |\underset{\sim}{B}| = \dfrac{\mu_0 I_0}{2\pi r}$

2.(i) 

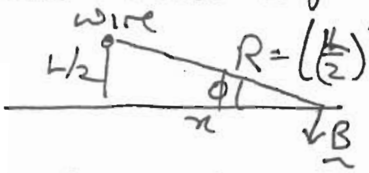
$r = |\underset{\sim}{r}| = \sqrt{R^2 + l^2}$ $\quad |d\underset{\sim}{l} \wedge \underset{\sim}{r}| = dl\, r \sin\theta = R\,dl$

$|d\underset{\sim}{B}| = \dfrac{\mu_0 I R\, dl}{4\pi (l^2+R^2)^{3/2}}$ $\quad |\underset{\sim}{B}| = \dfrac{\mu_0 I R}{4\pi} \displaystyle\int_{-L/2}^{L/2} \dfrac{dl}{(l^2+R^2)^{3/2}}$

always into page for all l.

Using given integral $|\underset{\sim}{B}| = \dfrac{\mu_0 I R}{4\pi} \dfrac{1}{R^2}\left[\dfrac{l}{(R^2+l^2)^{1/2}}\right]_{-L/2}^{L/2} = \dfrac{\mu_0 I}{2\pi R}\left(1 + \dfrac{4R^2}{L^2}\right)^{-1/2}$

(ii) Use the results from above for each side of square



$R = \left(\left(\frac{L}{2}\right)^2 + x^2\right)^{1/2}$ $\quad |\underset{\sim}{B}|$ due to top wire is $\dfrac{\mu_0 I}{2\pi R}\left(1 + \dfrac{L^2+4x^2}{L^2}\right)^{-1/2}$

Total $\underset{\sim}{B}$ due to all 4 sides must be along x-axis by symmetry

Total $|\underset{\sim}{B}|$ is $4 \times \dfrac{\mu_0 I}{2\pi R}\left(1 + \dfrac{L^2+4x^2}{L^2}\right)^{-1/2} \cdot \dfrac{L/2}{R}$ where $\sin\phi = \dfrac{L/2}{R}$

$|\underset{\sim}{B}| = \dfrac{2\sqrt{2}}{\pi} \mu_0 I L^2 (L^2+4x^2)^{-1} (L^2+2x^2)^{-1/2}$

(iii) Dipole moment is IL^2. On axis $\underset{\sim}{r} = x\underset{\sim}{i}$, $r = |\underset{\sim}{r}| = x$, $\underset{\sim}{M}_B \cdot \underset{\sim}{r} = IL^2 x$

Dipole field $\underset{\sim}{B} = \dfrac{\mu_0}{4\pi r^3}\left(\dfrac{3(\underset{\sim}{M}_B \cdot \underset{\sim}{r})\underset{\sim}{r}}{r^2} - \underset{\sim}{M}_B\right) \Rightarrow B_x = \dfrac{\mu_0}{4\pi x^3}\left(\dfrac{3IL^2x^2}{x^2} - 1\right)$ $\quad |\underset{\sim}{B}| = \dfrac{\mu_0 I L^2}{2\pi x^3}$

Expression from (ii) in limit $x \gg L$ (neglect L/x) is $|\underset{\sim}{B}| = \dfrac{2\sqrt{2}}{\pi}\mu_0 I L^2 \dfrac{4^{-1}\, 2^{-1/2}}{x^3} = \dfrac{\mu_0 I L^2}{2\pi x^3}$

$m\frac{d\underset{\sim}{v}}{dt} = q(\underset{\sim}{E} + \underset{\sim}{v}\wedge\underset{\sim}{B}) \Rightarrow m\frac{dv_x}{dt} = qv_yB$ (1), $m\frac{dv_y}{dt} = qE - qv_xB$ (2), $m\frac{dv_z}{dt} = 0$ (3)

(E: y dirⁿ, B: z dirⁿ)

Eqⁿ (3) $\Rightarrow$ $v_z = v_\parallel$ where $v_\parallel$ is a constant

Trial solⁿ $v_x = v_E + v_\perp \sin(\omega t + \psi) \Rightarrow \frac{dv_x}{dt} = \omega v_\perp \cos(\omega t + \psi)$

$v_y = v_\perp \cos(\omega t + \psi) \Rightarrow \frac{dv_y}{dt} = -\omega v_\perp \sin(\omega t + \psi)$

Subs into (1) $\Rightarrow$ $m\omega v_\perp \cos(\omega t+\psi) = qBv_\perp \cos(\omega t+\psi)$

Subs into (2) $\Rightarrow$ $-m\omega v_\perp \sin(\omega t+\psi) = qE - qv_EB - qv_\perp B\sin(\omega t+\psi)$

(1) + (2) are satisfied if $m\omega = qB$ and $E - v_EB = 0$.

$\Rightarrow \omega = qB/m$, $v_E = \frac{E}{B}$. ψ can take any value.

$B = 200T$, $E = 10^{10} Vm^{-1}$. Electron: negative charge so $q = -e$

Instead of using $\omega = \frac{qB}{m}$, define $\Omega = \frac{|e|B}{m}$ $(\omega \to -\Omega)$

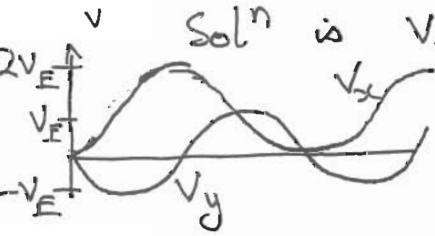
Solⁿ then $v_z = 0$ $v_x = v_E - v_\perp \sin(\Omega t + \phi)$ (3), $v_y = v_\perp\cos(\Omega t + \phi)$ (4)

where ϕ has replaced $-\psi$

$v_E = \frac{E}{B} = 5.10^7 ms^{-1}$. Eqⁿ (3) + (4) at $t=0$ $\Rightarrow$ $v_E - v_\perp \sin\phi = 0$, $v_\perp \cos\phi = 0$

For positive $v_\perp$, $\phi = \frac{\pi}{2}$, $v_\perp = v_E$

Solⁿ is $v_x = v_E(1 - \cos(\Omega t))$, $v_y = -v_E \sin(\Omega t)$



Integrate $\Rightarrow$ $x = v_E t - \frac{v_E}{\Omega}\sin\Omega t$, $y = \frac{v_E}{\Omega}\cos\Omega t$

Electron motion is a sum of a circular motion plus a drift at velocity v_E in x direction

$v_\perp = v_E = \frac{E}{B} = 5.10^7 ms^{-1}$, $\psi(= -\phi) = -\frac{\pi}{2}$, $\Omega = \frac{1.6.10^{-19}.200}{9.1.10^{-31}} = 3.5.10^{13}$ rad sec^{-1}

) Velocity v of charge carriers $|v| = \frac{E}{B}$, $E = \frac{0.81.10^{-6}}{1.5.10^{-2}} = 5.4.10^{-5} Vm^{-1}$

$v = |\underset{\sim}{v}| = \frac{5.4.10^{-5}}{0.4} = 1.35.10^{-4} ms^{-1}$.

Current $I = nvAe$ where n is density of carriers, e is their charge (electron)

A is cross section area of slab. $A = 2.10^{-3}.1.5.10^{-2} m^2 = 3.10^{-5} m^2$

$n = \frac{I}{vAe} = \frac{75}{1.35.10^{-4}.3.10^{-5}.1.6.10^{-19}} = 1.2.10^{29} m^{-3}$

Electrons pushed towards bottom of slab $\Rightarrow$ electric field is downwards

or (more simply) $E = -v\wedge B$ and v is in opposite dirⁿ to I.