Electricity & Magnetism, Problem Sheet 2 (Year 1)

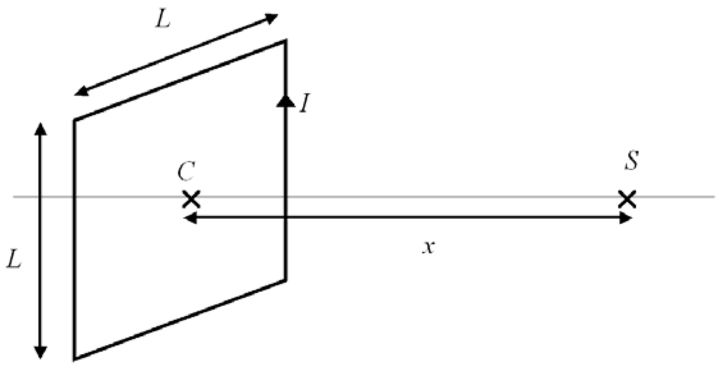
1) A wire with circular cross-section and radius R carries a current I_0. The current density is uniform throughout the wire. Show that the current carried at a radial distance r from the centre of the wire is $I(r)=I_0r^2/R^2$ for r<R. Use Ampere's law to find an expression for the magnetic field in the wire at radius r. Similarly find the magnetic field outside the wire at a radial distance r when r>R.

2) (i) A segment of wire of length L carries a current I. A point P is equidistant from the two ends of the wire and a distance R from the mid-point of the wire. Use the Biot-Savart law to show that the magnitude of the magnetic field at the point P is

$$B = \frac{\mu_0 I}{2\pi R}\left(1+\frac{4R^2}{L^2}\right)^{-1/2}$$

You may find it useful to know that $\int \frac{ds}{\left(s^2+a^2\right)^{3/2}} = \frac{1}{a^2}\frac{s}{\left(s^2+a^2\right)^{1/2}}$.

(ii)



S is a point a distance *x* from the centre *C* of a square loop with sides of length L carrying a current I as shown in the diagram. What is the magnitude of the magnetic field at the point *S*?

(iii) What is the dipole moment of the square loop in section (ii)? Use the equation $\mathbf{B} = (\mu_0/4\pi r^3)\,(3(\mathbf{M}_B.\mathbf{r})\mathbf{r}/r^2-\mathbf{M}_B)$ to obtain an approximate expression for the magnetic field at point S. Show that the approximate expression agrees with the expression for the magnetic field derived in section (ii) in the limit of large *x*.

NB *In the lectures I incorrectly wrote* $\mathbf{B} = (\mu_0/4\pi r^3)\,(\mathbf{M}_B - 3(\mathbf{M}_B.\mathbf{r})\mathbf{r}/r^2)$ *for the dipole field, ie I got the sign wrong. Please correct your notes.*

3) A particle with charge q and mass m is free to move in a uniform magnetic field **B** and a uniform electric field **E**. **B** is in the *z* direction and **E** is in the *y* direction. Show by substitution into the equations of motion for the particle that its velocity $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$ is given by

$$v_x = v_E + v_\perp \sin(\omega t + \psi) \qquad v_y = v_\perp \cos(\omega t + \psi) \qquad v_z = v_\parallel$$

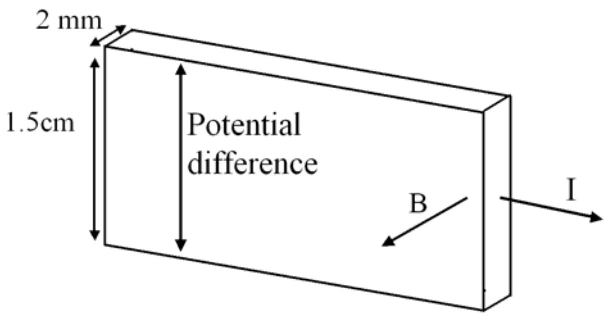
for a suitable choice of $v_\perp$, $v_\parallel$, v_E, ω, and ψ.

The interaction of a very powerful laser beam with a solid target produces strong magnetic and electric fields. Suppose the magnetic field is 200 Tesla (in the *z* direction) and the electric field is 10^{10} Vm^{-1} (in the *y* direction). An electron is at rest at *t*=0. What are the values of $v_\parallel, v_\perp$, v_E, ω, and ψ for the motion of the electron. Ignore relativistic corrections to the mass of the electron. Sketch v_x and v_y versus *t*. Describe the motion of the electron.

Mass of electron = 9.1×10^{-31} kg

Charge on an electron = -1.6×10^{-19} Coulomb

4)



A slab of copper, 2.0 mm thick and 1.5 cm wide, is placed in a uniform magnetic field with magnitude 0.4 Tesla. The field is normal to the face of the slab as shown in the figure. A current of 75 Amp is passed through the slab and this produces a potential difference of 0.81 μV across the slab. If the current is carried by electrons, in which direction is the electric field? Determine the number density of mobile electrons in the slab. (example given by Young & Freedman)