Problem Sheet 1   E+M yr 1

1) 

Pot^{l} $\phi = \dfrac{2Q}{2\pi\varepsilon_0 (x^2+a^2)^{1/2}}$ from principle of superposition

$$\phi = \frac{Q}{2\pi\varepsilon_0 a}\left(1+\frac{x^2}{a^2}\right)^{-1/2} \simeq \frac{Q}{2\pi\varepsilon_0 a}\left(1-\frac{1}{2}\frac{x^2}{a^2}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{2}{a}-\frac{x^2}{a^3}\right)$$

$E_x = -\dfrac{\partial\phi}{\partial x} = \dfrac{Qx}{2\pi\varepsilon_0 a^3}$ for $x \ll a$. By symmetry $E_y = 0$, $E_z = 0$ on axis.

$m\dfrac{d^2x}{dt^2} = qE_x = \dfrac{Qqx}{2\pi\varepsilon_0 a^3}$. If $Q \cdot q > 0$, body accelerates away from axis but acceleration decreases when x exceeds c

If Q & q have opposite charges, the body oscillates about the point $x = 0$ with frequency $\omega = \left(\dfrac{Qq}{2\pi\varepsilon_0 m a^3}\right)^{1/2}$. The body begins with potential energy $\dfrac{Qq}{2\pi\varepsilon_0 (a^2+b^2)^{1/2}}$. If $Q \cdot q > 0$, this is converted to kinetic energy and the final velocity is given by $\frac{1}{2}mv^2 =$ initial P.E.
So final vel $v = \left[\dfrac{Qq}{m\pi\varepsilon_0 (a^2+b^2)^{1/2}}\right]^{1/2}$.

2) $\underset{\sim}{M}\cdot\underset{\sim}{r} = M_x x + M_y y + M_z z \Rightarrow \phi = \dfrac{M_x x + M_y y + M_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$

$$\frac{\partial}{\partial x}\left[\frac{M_x x + M_y y + M_z z}{(x^2+y^2+z^2)^{3/2}}\right] = \frac{1}{(x^2+y^2+z^2)^{3/2}}\frac{\partial}{\partial x}\left[M_x x + M_y y + M_z z\right] + (M_x x + M_y y + M_z z)\frac{\partial}{\partial x}\left[\frac{1}{(x^2+y^2+z^2)^{3/2}}\right]$$

$$\frac{\partial\phi}{\partial x} = \left[\frac{M_x}{(x^2+y^2+z^2)^{3/2}} - \frac{3(M_x x + M_y y + M_z z)x}{(x^2+y^2+z^2)^{5/2}}\right]\frac{1}{4\pi\varepsilon_0}$$ Similarly for $\dfrac{\partial\phi}{\partial y}$, $\dfrac{\partial\phi}{\partial z}$

Comparing coordinates shows that $E = -\nabla\phi = \dfrac{3(\underset{\sim}{M}\cdot\underset{\sim}{r})\underset{\sim}{r} - \underset{\sim}{M}r^2}{4\pi\varepsilon_0 r^5}$

3) Gauss's flux law $\Rightarrow 4\pi r^2 E =$ enclosed charge$/\varepsilon_0$

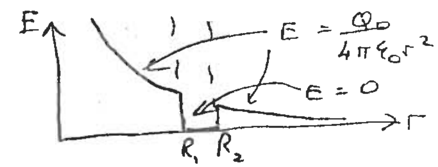
$r < R_1$ $4\pi r^2 E = Q_0/\varepsilon_0 \Rightarrow E = Q_0/4\pi\varepsilon_0 r^2$

$R_1 < r < R_2$ $4\pi r^2 E = (Q_0 + Q_1)/\varepsilon_0 \Rightarrow E = (Q_0+Q_1)/4\pi\varepsilon_0 r^2$

$r > R_2$ $4\pi r^2 E = (Q_0 + Q_1 + Q_2)/\varepsilon_0 \Rightarrow E = (Q_0+Q_1+Q_2)/4\pi\varepsilon_0 r^2$

$E = 0$ inside a conductor, so $Q_1 = -Q_0$

If $Q_1 = -Q_0$ and $Q_2 = Q_0$



For $r > R_2$ $\phi = -\displaystyle\int_\infty^r E\,dr = \dfrac{Q_0}{4\pi\varepsilon_0 r}$

For $R_1 < r < R_2$ ϕ is constant so $\phi = \dfrac{Q_0}{4\pi\varepsilon_0 R_2}$

For $r < R_1$ $\phi = \dfrac{Q_0}{4\pi\varepsilon_0 R_2} - \displaystyle\int_r^{R_1}\frac{Q_0}{4\pi\varepsilon_0 r^2}\,dr = \dfrac{Q_0}{4\pi\varepsilon_0}\left(\dfrac{1}{R_2} + \dfrac{1}{r} - \dfrac{1}{R_1}\right)$