Electricity & Magnetism, Problem Sheet 1 (Year 1)

1) Two identical charges Q are fixed and separated by a distance 2a on the y-axis. Their positions are (0,a,0) and (0,-a,0) respectively. What is the electric potential φ at the position (x,0,0) on the x-axis.

If x<<a, use a Taylor expansion to show that

$$\varphi = \frac{Q}{4\pi\varepsilon_0}\left(\frac{2}{a} - \frac{x^2}{a^3}\right) \qquad (1)$$

when terms proportional to higher powers of x are neglected.

Derive an expression for the electric field at position (x,0,0) for x<<a.

A body with mass m and charge q is free to move along the x axis. Show that the equation of motion of the body is

$$m\frac{d^2x}{dt^2} = \frac{Qqx}{2\pi\varepsilon_0 a^3} \qquad (2)$$

The body is initially at rest at x = b (b<<a). Describe the subsequent motion of the body if (i) both Q and q are positive (ii) Q and q have opposite signs. Does x become larger than a in either case? If so, does equation (2) become invalid, and what is the final velocity of the body?

2) The potential at position **r** due to an electric dipole **M** is

$$\varphi(\mathbf{r}) = \frac{\mathbf{M}.\mathbf{r}}{4\pi\varepsilon_0 r^3}.$$

If $\mathbf{M}=M_x\mathbf{i}+M_y\mathbf{j}+M_z\mathbf{k}$ and $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, satisfy yourself that the potential can be written in the form

$$\varphi = \frac{M_x x + M_y y + M_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

Show that

$$\frac{\partial\varphi}{\partial x} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{M_x}{(x^2+y^2+z^2)^{3/2}} - \frac{3(M_x x + M_y y + M_z z)x}{(x^2+y^2+z^2)^{5/2}}\right\}$$

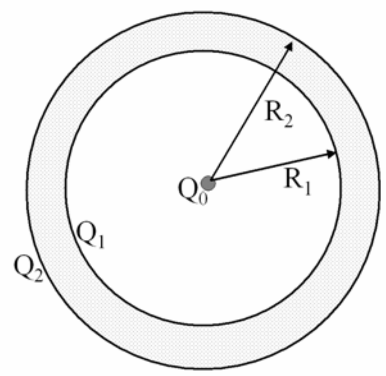
$$\frac{\partial\varphi}{\partial y} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{M_y}{(x^2+y^2+z^2)^{3/2}} - \frac{3(M_x x + M_y y + M_z z)y}{(x^2+y^2+z^2)^{5/2}}\right\}$$

$$\frac{\partial\varphi}{\partial z} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{M_z}{(x^2+y^2+z^2)^{3/2}} - \frac{3(M_x x + M_y y + M_z z)z}{(x^2+y^2+z^2)^{5/2}}\right\}$$

Hence show that the electric field at position **r** due to the dipole is given by

$$\mathbf{E} = \frac{3(\mathbf{M}.\mathbf{r})\mathbf{r} - \mathbf{M}r^2}{4\pi\varepsilon_0 r^5}$$

3) A point charge Q_0 lies at the centre of a conducting spherical shell. The inner radius of the conducting shell is R_1 and the outer radius is R_2.



Suppose that a charge Q_1 is uniformly distributed over the inner surface of the shell and a charge Q_2 is uniformly distributed over the outer surface of the shell. Derive expressions for the electric field (i) inside the shell ($r<R_1$) (ii) within the conductor ($R_1<r<R_2$), and (iii) outside the shell ($r>R_2$), where r is the radial distance from the charge Q_0.

The electric field has to be zero inside a conducting medium. What does this tell us about the relationship between Q_0 and Q_1?

If $Q_2 = Q_0$, $Q_1 = -Q_0$, and Q_0 is positive, sketch a graph of electric field as a function of radius r.

Also for $Q_2 = Q_0$, $Q_1 = -Q_0$, derive the potential as a function of r and sketch the potential.

Tony Bell, Feb 2005