classwork 5 E+M yr1 (10 March 2005)

1) $m\frac{d\mathbf{v}}{dt} = q\,\mathbf{v}\wedge\mathbf{B} \Rightarrow \frac{dv_x}{dt} = \frac{q}{m}v_y B$ (1), $\frac{dv_y}{dt} = -\frac{qv_x}{m}B$ (2), $\frac{dv_z}{dt} = 0$ (3)

satisfied by $v_z = v_{\parallel}$ (constant)

$v_x = v_\perp \sin(\omega t+\psi) \Rightarrow \frac{dv_x}{dt} = \omega v_\perp \cos(\omega t+\psi)$

$v_y = v_\perp \cos(\omega t+\psi) \Rightarrow \frac{dv_y}{dt} = -\omega v_\perp \sin(\omega t+\psi)$

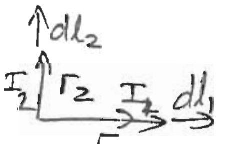
Subs into eq^{n}s of motion (1) $\Rightarrow \omega v_\perp \cos(\omega t+\psi) = \frac{q}{m}Bv_\perp \cos(\omega t+\psi)$

(2) $\Rightarrow -\omega v_\perp \sin(\omega t+\psi) = -\frac{qB}{m}v_\perp \sin(\omega t+\psi)$ ← Both satisfied if $\omega = \frac{qB}{m}$

For any $v_\perp$ and ψ.

$B = 0.5$ Tesla $\Rightarrow \omega = \frac{1.6\cdot 10^{-19}\cdot 0.5}{9.11\cdot 10^{-31}} = 8.8\cdot 10^{10}\,s^{-1}$

Number of rotations in 1 sec is $\omega/2\pi = 1.4\cdot 10^{10}$.



Force on dl_2 is $\mathbf{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi}\frac{d\mathbf{l}_2\wedge[d\mathbf{l}_1\wedge(\mathbf{r}_2-\mathbf{r}_1)]}{|\mathbf{r}_2-\mathbf{r}_1|^3}$

Force on dl_1 is $\mathbf{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi}\frac{d\mathbf{l}_1\wedge[d\mathbf{l}_2\wedge(\mathbf{r}_1-\mathbf{r}_2)]}{|\mathbf{r}_1-\mathbf{r}_2|^3}$

$\mathbf{r}_1 = 30\mathbf{i}$ $\mathbf{r}_2 = 40\mathbf{j}$ $d\mathbf{l}_1 = 0.002\mathbf{i}$ $d\mathbf{l}_2 = 0.004\mathbf{j}$ $\mathbf{r}_1-\mathbf{r}_2 = 30\mathbf{i}-40\mathbf{j}$

$|\mathbf{r}_1-\mathbf{r}_2| = |\mathbf{r}_2-\mathbf{r}_1| = \sqrt{30^2+40^2} = 50$ $I_1 = I_2 = 1$ Amp $\mu_0 = 4\pi\cdot 10^{-7}\,Hm^{-1}$

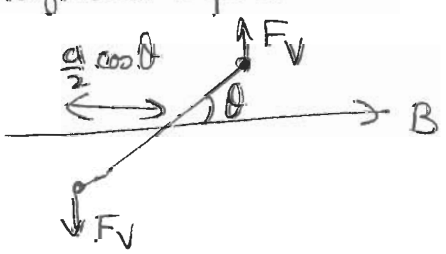
$d\mathbf{l}_1\wedge(\mathbf{r}_2-\mathbf{r}_1) = 0.002\mathbf{i}\wedge(40\mathbf{j}-30\mathbf{i}) = 0.08\mathbf{k}$; $d\mathbf{l}_2\wedge[d\mathbf{l}_1\wedge(\mathbf{r}_2-\mathbf{r}_1)] = 0.004\mathbf{j}\wedge 0.08\mathbf{k} = 3.2\cdot 10^{-4}\mathbf{i}$

$d\mathbf{l}_2\wedge(\mathbf{r}_1-\mathbf{r}_2) = 0.004\mathbf{j}\wedge(30\mathbf{i}-40\mathbf{j}) = 0.12\mathbf{k}$; $d\mathbf{l}_1\wedge[d\mathbf{l}_2\wedge(\mathbf{r}_1-\mathbf{r}_2)] = 0.002\mathbf{i}\wedge(-0.12)\mathbf{k} = 2.4\cdot 10^{-4}\mathbf{j}$

$\mathbf{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi}\cdot\frac{3.2\cdot 10^{-4}}{50^3}\mathbf{i} = 2.6\cdot 10^{-16}\mathbf{i}\,N$ $\mathbf{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi}\cdot\frac{2.4\cdot 10^{-4}}{50^3}\mathbf{j} = 1.9\cdot 10^{-16}\mathbf{j}\,N$

Action not equal to reaction because complete circuit needs to be considered. In isolation, this is an impossible configuration.

3) Force $F = I\,\mathbf{l}\wedge\mathbf{B}$. Force on top segment $F_{top} = aIB\sin\theta$.
Force on bottom segment $F_{bot} = aIB\sin\theta$. Force on each vertical segment is $F_V = aIB$ since wire and B are perpendicular.
Magnetic dipole moment $|M_B| = IA = Ia^2$



Torque $= 2\times\left(\frac{a}{2}\cos\theta\right)\times aIB = a^2IB\cos\theta$
$(\mathbf{T} = \mathbf{r}\wedge\mathbf{F})$

Comes to rest at $\theta = \frac{\pi}{2}$