Classwork 4 E+M yr 1 3rd March 2005

1) $\underset{\sim}{B}_{ex} = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$ ↑ exact

$\underset{\sim}{B}_{dip} = \frac{\mu_0 I}{4\pi r^3}\left(\underset{\sim}{M}_B - 3\frac{(\underset{\sim}{M}_B \cdot \underset{\sim}{r})\underset{\sim}{r}}{r^2}\right)$ ↑ dipole $\quad \underset{\sim}{M}_B = \pi a^2 \underset{\sim}{i}$

On axis $\underset{\sim}{r} = x\underset{\sim}{i}$, $\underset{\sim}{M}_B = M_B \underset{\sim}{i} \Rightarrow B_{dip} = \frac{\mu_0 I}{4\pi x^3}\left(M_B \underset{\sim}{i} - 3\frac{(M_B x)x}{x^2}\underset{\sim}{i}\right)$

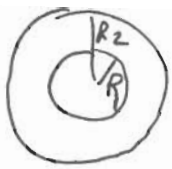
$|B_{dip}| = \frac{\mu_0 I}{4\pi x^3}|(-2M_B)| = \frac{\mu_0 I a^2}{2x^3}$ (i)

(ii) $\frac{B_{dip}}{B_{ex}} = \frac{(x^2+a^2)^{3/2}}{x^3} = \left(1+\frac{a^2}{x^2}\right)^{3/2}$

$\frac{B_{dip}}{B_{ex}} = 1.1 \Rightarrow \left(1+\frac{a^2}{x^2}\right)^{3/2} = 1.1 \Rightarrow \frac{a^2}{x^2} = 1.1^{2/3} - 1 \Rightarrow \frac{x}{a} = \frac{1}{(1.1^{2/3}-1)^{1/2}} = 3.9$

For agreement to better than 10%, $x > 3.9a$

2)



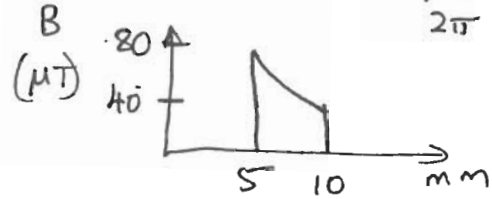
$\int \underset{\sim}{B}.d\underset{\sim}{l} = \mu_0 I \Rightarrow 2\pi r B =$ $\mu_0 \times$ contained current

$r < R_1$, no current inside path $\Rightarrow B = 0$

$R_1 < r < R_2$, current inside path is $I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

$r > R_2$, currents balance to zero $\Rightarrow B = 0$

$I = 2$, for $R_1 < r < R_2$ $B = \frac{\mu_0 . 2}{2\pi r} = 4.10^{-7} r^{-1}$ ↑ in mm, $B = 4.10^{-4}\left(\frac{r}{mm}\right)^{-1}$ T



$r = 5$mm $B = 80\ \mu T$

$r = 10$mm $B = 40\ \mu T$

3) Energy (rest mass) of proton = $m_p c^2 = 1.67.10^{-27}.9.10^{16} = 1.50.10^{-10}$ J $= 939$ MeV

Lorentz factor = $\frac{3.10^{20}\text{eV}}{9.39.10^8\text{eV}} = 3.19.10^{11}$ Rel. mass $(\gamma m_p) = 5.33.10^{-16}$ kg

Moves at speed of light so momentum $(\gamma m_p c) = 1.6.10^{-7}$ kgms^{-1}

Max Force = $ecB = 1.6.10^{-19}.3.10^8.10^{-13} = 4.8.10^{-24}$ N

Time of flight = $\frac{20Mpc}{c} = \frac{20 \times 3.1.10^{22}}{3.10^8} = 2.1.10^{15}$ sec

Max change of momentum = $4.8.10^{-24}.2.1.10^{15} = 10^{-8}$ kg ms^{-1}

change of mmtm much smaller (6%) than actual mmtm

Direction of trajectory not change much → good idea of origin

If mag. field disordered over Mpc distance, total deviation smaller