1st year Electricity and Magnetism, Tony Bell

Classwork 4 – 3rd March 2005

1) In the lectures (section E.4) we derived an exact expression for the magnitude of the magnetic field $|\mathbf{B}| = \mu_0 I a^2/2(x^2+a^2)^{3/2}$ on the axis at a distance x from a circular current loop of radius a carrying a current I . Section E.5 gave an expression for the magnetic field in the dipole approximation $\mathbf{B} = (\mu_0/4\pi r^3)\,(\mathbf{M}_B - 3(\mathbf{M}_B.\mathbf{r})\mathbf{r}/r^2)$ at a position $\mathbf{r}$ relative to a loop with dipole moment $\mathbf{M}_B$ ($|\mathbf{M}_B| = I\pi a^2$ for a loop of radius a carrying a current I).

(i) Sketch a graph of $|\mathbf{B}|$ against distance x from the loop giving curves for both the exact value and the value in the dipole approximation.
(ii) How large must x be for the dipole and exact values to agree to better than 10%?

2) A coaxial cable consists of two thin cylinders of radius R_1 and R_2 respectively ($R_2>R_1$). The inner cylinder carries a current I in the positive x direction. The outer cylinder carries a current with the same magnitude I but in the opposite (-x) direction. Use Ampere's law to derive the magnetic field a distance r from the centres of the cylinders for (i) $r<R_1$ (ii) $R_1<r<R_2$ (iii) $r>R_2$. Sketch a plot of magnetic field versus radius r for the case in which I=2Amp, R_1=5mm and R_2=10mm. ($\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1})

3) The highest energy cosmic rays arriving at the earth have energies of 3×10^{20}eV. Assuming that these cosmic rays are protons, calculate their Lorentz factor, their relativistic mass (in kg), and their momentum. They must have travelled distances less than about 20Mpc, otherwise they would lose energy by interacting with the cosmic microwave background. If the magnetic field between galaxies is typically 10^{-13} Tesla, can their direction of arrival be expected to give information about the origin of high energy cosmic rays?

1eV $=1.6\times10^{-19}$J
Mass of proton = 1.67×10^{-27} kg
Charge on a proton = 1.6×10^{-19} Coulomb
Speed of light = 3×10^{8} ms^{-1}
Force on a proton moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is $e\mathbf{v}$^$\mathbf{B}$
1 Mpc = 3.1×10^{22} m