E & M Classwork 3 year 1 24th Feb 2005

- $C = A\varepsilon_0/x = 10^{-3}.\ 8.854.10^{-12}/10^{-3} = 8.854\ \text{pF}$

$Q = 10^{-11}$ Coulomb. $V = Q/C = 10^{-11}/8.854.10^{-12} = 1.13\text{V}$

E = p.d./separation $= 1.1/10^{-3} = 1.13.10^{3}\ \text{Vm}^{-1}$

Energy $= \frac{1}{2}Q^2/C = \frac{1}{2}.10^{-22}/8.854.10^{-12} = 5.6.10^{-12}\ \text{J}$

Force $= \frac{1}{2}QE = \frac{1}{2}.10^{-11}.\ 1.13.10^{3} = 5.6.10^{-9}\ \text{N}$

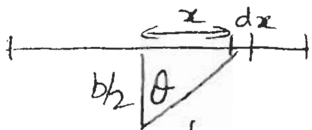
Electrical energy density $\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}.8.854.10^{-12}.\ 1.13^2 \times 10^6 = 5.6.10^{-6}\ \text{Jm}^{-3}$

$\frac{1}{2}\varepsilon_0 E^2 \times$ volume $= 5.6.10^{-6}.\ 10^{-3}.10^{-3} = 5.6\ 10^{-12}\ \text{J}$

2. $\underset{\sim}{B} = \dfrac{\mu_0 I\ d\underset{\sim}{l} \wedge \underset{\sim}{r}}{4\pi r^3}$ $\underset{\sim}{r} = \underset{\sim}{r}_2 - \underset{\sim}{r}_1 = -3\underset{\sim}{i} - 4\underset{\sim}{k}$ $|\underset{\sim}{r}| = \sqrt{3^2+4^2} = 5$

$$d\underset{\sim}{l} \wedge \underset{\sim}{r} = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ -3 & 0 & -4 \end{vmatrix} = 2j \Rightarrow \underset{\sim}{B} = \frac{4\pi.10^{-7}.\ 3.\ 2j}{4\pi\ 5^3} = 4.8j\ \text{nT}$$

3. (i)



Calculate field due to one side. Field is normal to paper and in same direction for each element dx. $dB = \dfrac{\mu_0 I}{4\pi}\dfrac{dx}{(x^2 + (\frac{b}{2})^2)}\cos\theta$

Integrate along wire $B_1 = \displaystyle\int_{-b/2}^{b/2} \frac{\mu_0 I}{4\pi}\frac{b/2\ dx}{(x^2+(\frac{b}{2})^2)^{3/2}}$

Subs $s = \dfrac{x}{b/2}$ $ds = \dfrac{2dx}{b}$ $B_1 = \dfrac{\mu_0 I}{2\pi}\displaystyle\int_{-1}^{1}\frac{ds}{(s^2+1)^{3/2}} = \frac{\mu_0 I}{2\pi b}\left[\frac{s}{(1+s^2)^{1/2}}\right]_{-1}^{1}$

Contributions from each side add in same direction $= \dfrac{\mu_0 I}{\pi b\sqrt{2}}$

so total field is $\dfrac{\mu_0 I\, 2\sqrt{2}}{\pi b}$. $I = 0.5\text{A}, b = 0.2\text{m} \Rightarrow B = \dfrac{4\pi.10^{-7}.0.5.2\sqrt{2}}{\pi\ 0.2} = 2.8\mu\text{T}$

(ii) Dipole moment is current × area so $M_B = Ib^2 = 0.5.0.2^2 = 0.02\ \text{Am}^2$

Given in lectures: $B = \dfrac{\mu_0}{4\pi r^3}\left(\underset{\sim}{M}_B - \dfrac{3(\underset{\sim}{M}_B.\underset{\sim}{r})\underset{\sim}{r}}{r^2}\right)$

On the axis M_B and r are in the same direction so $|\underset{\sim}{B}| = \dfrac{2\mu_0 M_B}{4\pi r^3}$

$$B = \frac{2.\ 4\pi.\ 10^{-7}\ 0.02}{4\pi.\ 10^3} = 4.10^{-12}\ \text{T}$$