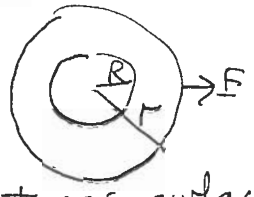
Classwork 2, EM yr 1, 17th Feb 2005



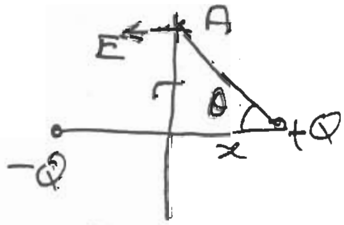
1 (i). Apply Gauss's law to cylinder of length l and radius r concentric with charge cylinder.
By symmetry, E is radial so $\int \underset{\sim}{E} \cdot d\hat{A}$ equals E times surface area of integration cylinder $\int \underset{\sim}{E} \cdot d\hat{A} = E \cdot 2\pi r l$.
Charge inside cylinder $Q = \rho l \Rightarrow 2\pi r l E = \rho l / \varepsilon_0 \Rightarrow E = \rho / 2\pi \varepsilon_0 r$

If integration surface inside charge cylinder $(r<R)$ there is no charge inside integration cylinder $(Q=0)$, so $E=0$

(ii) In this case the field outside $(r>R)$ is same as in (i).
If $r<R$, charge inside integration cylinder is $Q = \rho l \left(\frac{r}{R}\right)^2$.
If $r<R$, $2\pi r l E = \frac{1}{\varepsilon_0} \rho l \left(\frac{r}{R}\right)^2 \Rightarrow E = \rho r / 2\pi \varepsilon_0 R^2$.

2. σ is the charge per unit area. The electric field is normal to the conductor. The extra factor of 2 comes from the fact that there is an electric on both sides of the sheet. In comparison, $E=0$ inside a conductor giving a contribution to $\int \underset{\sim}{E} \cdot d\hat{A}$ from only one side.

3. By method of images, an equivalent configuration is



The electric field due to $+Q$ at A is

$E = \frac{Q}{4\pi\varepsilon_0 (r^2+x^2)}$. The component normal to the conductor is $E_\perp = E\cos\theta = \frac{Qx}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$

The component in the r dir^n is $E_r = E\sin\theta$.
The contribution from $-Q$ is the same except that E_r acts in the opposite directions and cancels out the E_r from $+Q$.
The result is an electric field normal to the conductor of magnitude

$$\frac{Qx}{2\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

From $E = \sigma/\varepsilon_0$ $\quad \sigma = \frac{-Qx}{2\pi (r^2+x^2)^{3/2}}$ The minus sign comes from the direction of E.

To find total charge on conductor, divide into rings of radius r and thickness dr, each carrying a charge $2\pi r dr \sigma$
Total charge on conductor is $\int_0^\infty 2\pi r dr \sigma = -\int_0^\infty \frac{x r Q dr}{(r^2+x^2)^{3/2}} = -Q$