Electricity & Magnetism, yr 1, Classwork 1

1 a) Dipole moment $\underset{\sim}{M}_1 = Qa\,\hat{\underset{\sim}{x}}$

b) $\phi = \dfrac{\underset{\sim}{M}_1 \cdot \underset{\sim}{r}}{4\pi\varepsilon_0 r^3}$. Along x axis $\underset{\sim}{M}_1$ and $\underset{\sim}{r}$ are in same dirⁿ so $\underset{\sim}{M}_1 \cdot \hat{\underset{\sim}{r}} = M_1 X$

$\phi = \dfrac{M_1}{4\pi\varepsilon_0 X^2}$. Electric field is in x dirⁿ by symmetry

$E_x = -\dfrac{\partial\phi}{\partial X} = \dfrac{M_1}{2\pi\varepsilon_0 X^3}$

c) $\phi\left(X \pm \frac{b}{2}\right) = \dfrac{M_1}{4\pi\varepsilon_0\left(X\pm\frac{b}{2}\right)^2}$, $E_x\left(X+\frac{b}{2}\right) = \dfrac{M_1}{2\pi\varepsilon_0\left(X\pm\frac{b}{2}\right)^3}$

d) Total force $F_x = qE_x\left(X+\frac{b}{2}\right) - qE_x\left(X-\frac{b}{2}\right) = \dfrac{M_1 q}{2\pi\varepsilon_0}\left[\dfrac{1}{\left(X+\frac{b}{2}\right)^3} - \dfrac{1}{\left(X-\frac{b}{2}\right)^3}\right]$

Taylor expansion neglecting b^2 terms $F_x = \dfrac{M_1 q}{2\pi\varepsilon_0}\dfrac{1}{X^3}\left[1 - \dfrac{3b}{2X} - \left(1+\dfrac{3b}{2X}\right)\right] = -\dfrac{3M_1 qb}{2\pi\varepsilon_0 X^4}$

$F_x = -\dfrac{3M_1M_2}{2\pi\varepsilon_0 X^4}$ F_x is negative so it is attractive ($-x$ dirⁿ)

e) $\text{P.E.} = q\phi\left(X+\frac{b}{2}\right) - q\phi\left(X-\frac{b}{2}\right) = \dfrac{M_1 q}{4\pi\varepsilon_0}\left[\dfrac{1}{\left(X+\frac{b}{2}\right)^2} - \dfrac{1}{\left(X-\frac{b}{2}\right)^2}\right]$

$= -\dfrac{M_1 qb}{2\pi\varepsilon_0 X^3}$ in limit of small b. $\text{P.E.} = -\dfrac{M_1M_2}{2\pi\varepsilon_0 X^3}$

From b) $E_x = \dfrac{M_1}{2\pi\varepsilon_0 X^3} \Rightarrow \text{P.E.} = -E_x M_2$

E_x and M_2 are both components in x dirⁿ so $\text{P.E.} = -\underset{\sim}{E}\cdot\underset{\sim}{M}_2$

f) Rotating either of the dipoles through 180° is like changing sign of M_1 or M_2. So P.E. changes from $-\dfrac{M_1M_2}{2\pi\varepsilon_0 X^3}$ to $+\dfrac{M_1M_2}{2\pi\varepsilon_0 X^3}$

Energy required is $\dfrac{M_1M_2}{\pi\varepsilon_0 X^3}$

2. Electric field due to 1st dipole is $E_x = \dfrac{M_1}{2\pi\varepsilon_0 X^3}$

If 2nd dipole is α times E_x, $M_2 = \dfrac{\alpha M_1}{2\pi\varepsilon_0 X^3}$

From 1d), $F_x = -\dfrac{3M_1M_2}{2\pi\varepsilon_0 X^4} = -\dfrac{3}{2\pi\varepsilon_0 X^4} \cdot M \cdot \dfrac{\alpha M_1}{2\pi\varepsilon_0 X^3} \propto \dfrac{1}{X^7}$

Force is proportional to inverse 7th power of separation

Assuming $\alpha > 0$, F_x is negative $\Rightarrow$ attractive

α likely to be greater than zero because positive electric field moves positive charge away from 1st dipole. Another way of looking at it is that the P.E. is negative in this configuration.