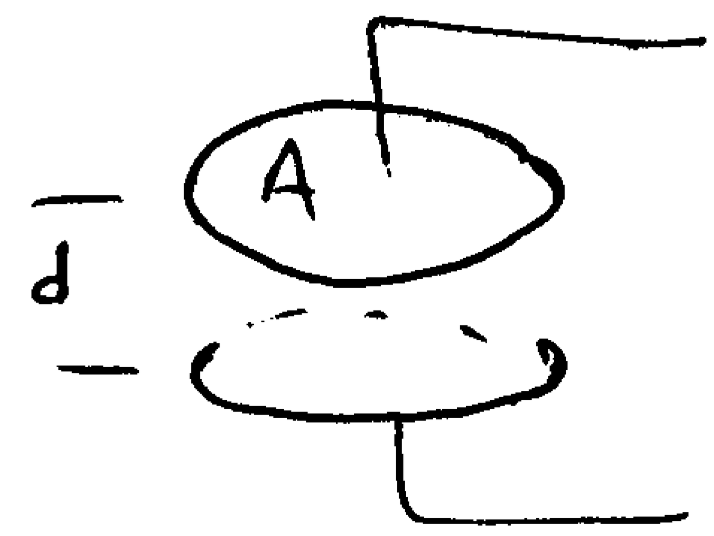
Solutions - Electricity & Magnetism CW5

1) For a parallel plate capacitor,

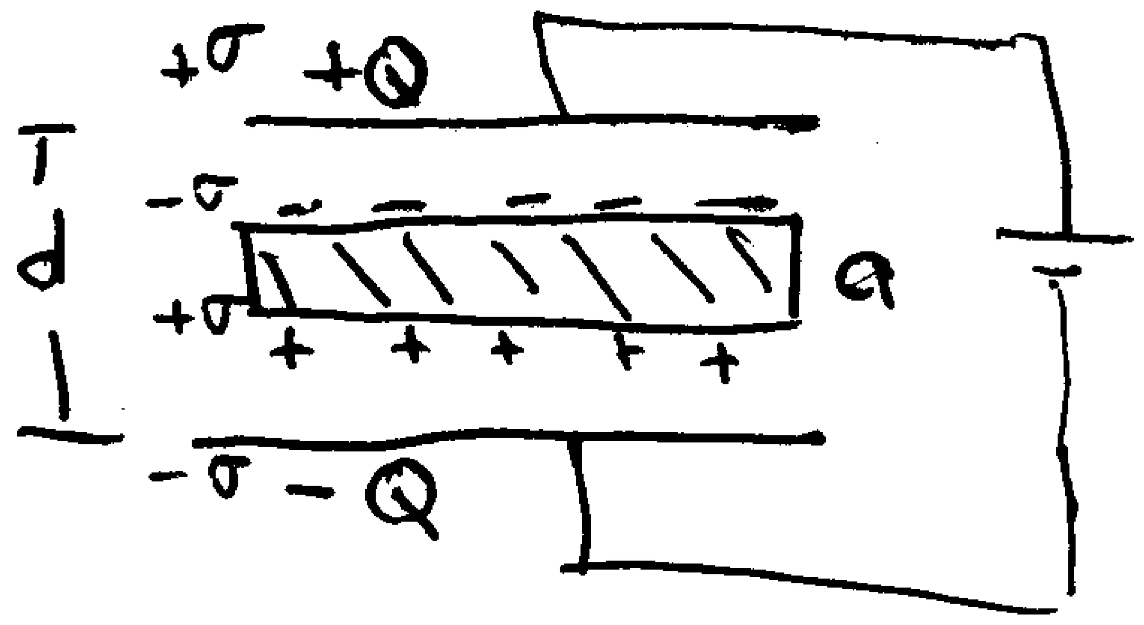
$$C = \epsilon_0 A / d \quad \text{so} \quad d = \frac{\epsilon_0 A}{C}$$

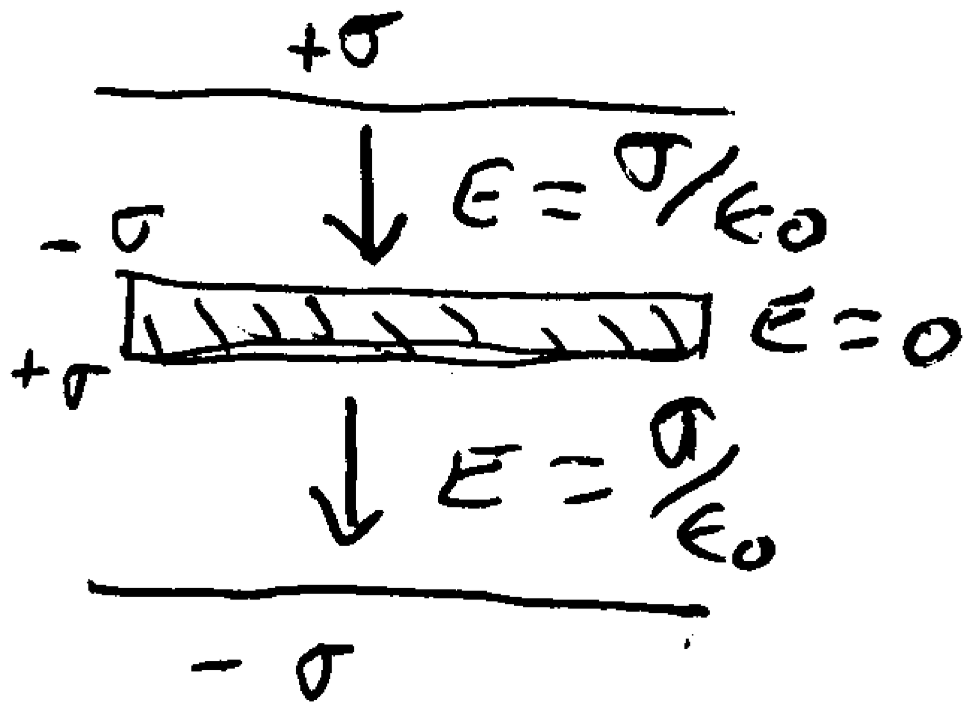


Putting in numbers $d = \dfrac{8.85\times10^{-12}\,\pi(0.01)^2}{10^{-12}} = 2.78\times10^{-3}\,\mathrm{m} = \underline{2.78\,\mathrm{mm}}$

This is just over $\frac{1}{10}$ th the diameter of the coin, so the configuration looks like  and the infinite sheet assumption (neglecting edge effects) should be just about ok.

2) a) Charges in the conductor will move to make $\underset{\sim}{E} = \underset{\sim}{0}$ within the conductor. This requires the same charge/area to appear there as on the plates of the capacitor, ie $\sigma = Q/A$. Then we find the voltage between the plates by integrating from one to the other.





This gives $V = E(d-a) = \dfrac{\sigma(d-a)}{\epsilon_0} = \dfrac{Q(d-a)}{A\,\epsilon_0}$

Since $C = \dfrac{Q}{V}$ we find $\underline{C = \dfrac{A\epsilon_0}{d-a}}$

b) Re-write as $C = \dfrac{A\epsilon_0}{d}\,\dfrac{1}{1-a/d} \equiv C_0\,\dfrac{1}{1-a/d}$

c) For $a \to 0$ (no metal slab) $C \to C_0$ which makes sense
For $a \to d$ $C \to \infty$ as $V \to 0$. The region where $E \neq 0$ vanishes and putting more charge on the plates merely causes more charge separation within the conductor.

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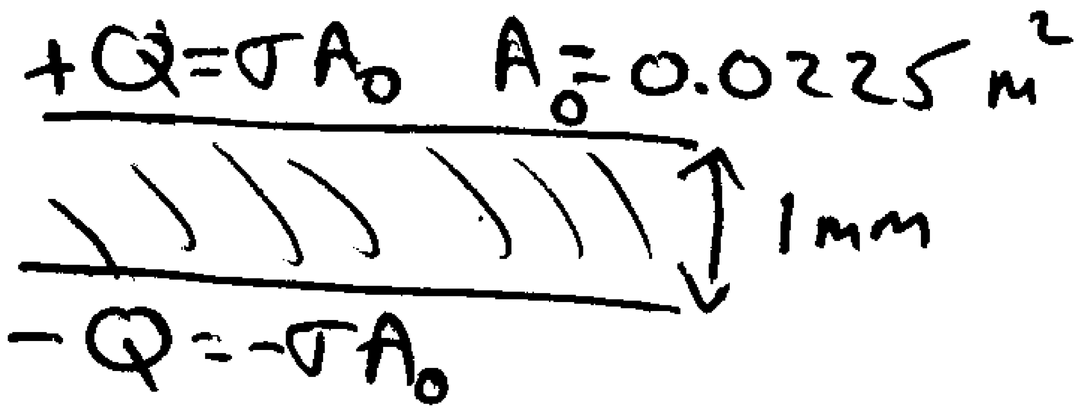
(3) a) $C = Q/V \Rightarrow V = Q/C$ & $\underline{Q = CV}$



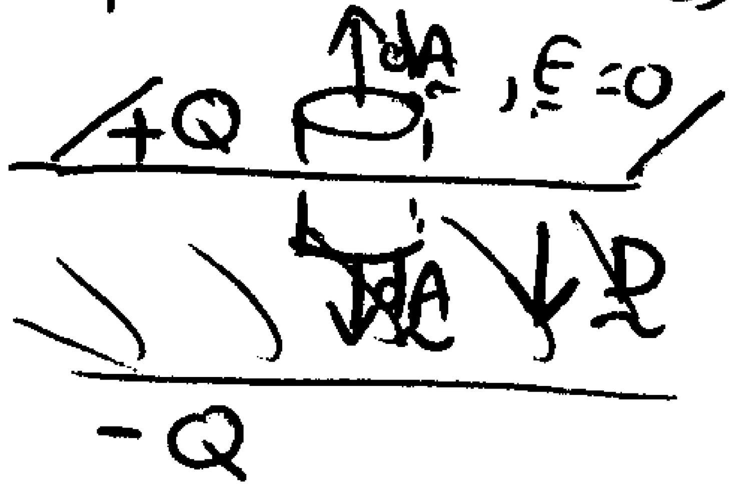
For a capacitor filled with dielectric

$$C = \epsilon_r C_0 = \epsilon_r \epsilon_0 \frac{A_0}{d} \Rightarrow Q = \left(\epsilon_r \epsilon_0 \frac{A_0}{d}\right) V$$

Putting in numbers:

$$Q = 2.1 \times 8.85 \times 10^{-12} \frac{0.0225}{1 \times 10^{-3}} \times 12 = \underline{5.02 \times 10^{-9} \text{ Coulombs}}$$

b) We know $\underset{\sim}{D}$ outside $= 0$, from previous work or, e.g., considering separately the two sides. Put a pillbox ("Gaussian") across one of the plates & into the Teflon. Then Gauss gives:



$$\oiint \underset{\sim}{D} \cdot d\underset{\sim}{A} = \underbrace{\iint}_{top\ 0} + \iint_{bottom} + \underbrace{\iint}_{sides\ 0}$$

$$= +DA = Q_{encl\ free} = \sigma_{free} A \Rightarrow \epsilon_r \epsilon_0 E = \frac{Q}{A_0}$$

Thus $E = \dfrac{Q}{A_0 \epsilon_r \epsilon_0} = \dfrac{(\cancel{\epsilon_r \epsilon_0 A_0} / d)\, V}{\cancel{A_0 \epsilon_r \epsilon_0}} = \dfrac{V}{d} = \dfrac{12}{1 \times 10^{-3}} = \underline{\underline{1.2 \times 10^4 \ \tfrac{V}{m}}}$

c) Disconnect V and then remove Teflon. So we retain same Q.

Repeat Gauss with $\epsilon_r = 1$ to give $E = \dfrac{Q}{A_0 \epsilon_0}$

and so

$$E = \frac{5.02 \times 10^{-9}}{0.0225\ 8.85 \times 10^{-12}} = 2.52 \times 10^4 \frac{V}{M}$$

$(= \epsilon_{r\,teflon} \times$ result in (b) $)$

④ For v_i southward, the initial $v_i \times B$ is to the East.

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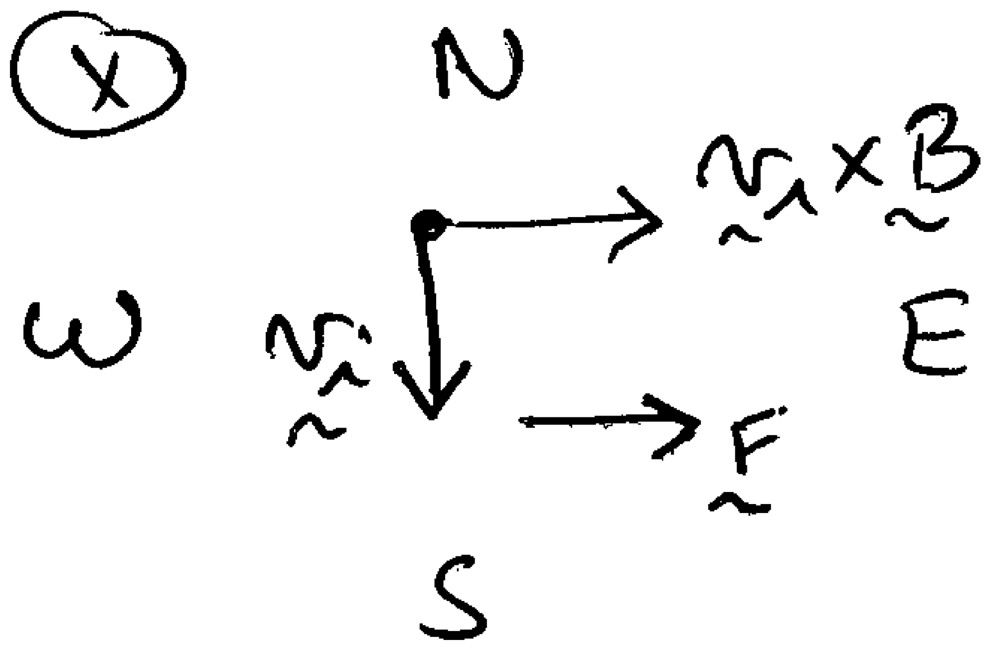

The magnetic force is $F = q\, v \times B$
and since the deflection is Eastward, parallel to $v_i \times B$,
we must have $\underline{\underline{q > 0}}$

⑤ $q = -1.24 \times 10^{-8}\,C$, $v = (4.19 \times 10^4, -3.85 \times 10^4, 0)$

Lorentz force $F = q\, v \times B$ so

a) $B = 1.4\,T\,\hat{i}$: $F = -1.24 \times 10^{-8} (4.19, -3.85, 0) 10^4 \times 1.4\hat{i}$

$= -1.24 \times 10^{-8} (-3.85 \times 10^4) \times 1.4 \times (\hat{j} \times \hat{i})$

ie $F = -6.68 \times 10^{-4}\,N\,\hat{k}$ $\quad [\hat{j} \times \hat{i} = -\hat{k}]$

b) $B = 1.4\,T\,\hat{k}$:

$F = -1.24 \times 10^{-8} \times 10^4 (4.19, -3.85, 0) \times (0, 0, 1.4)$

$= -1.24 \times 10^{-4} \left[(4.19)(1.4)\,\hat{i} \times \hat{k} + (-3.85)(1.4)\,\hat{j} \times \hat{k}\right]$

$= (+7.27 \times 10^{-4}\,\hat{j} + 6.68 \times 10^{-4}\,\hat{i})\ N$ $\quad [\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{k} = +\hat{i}]$