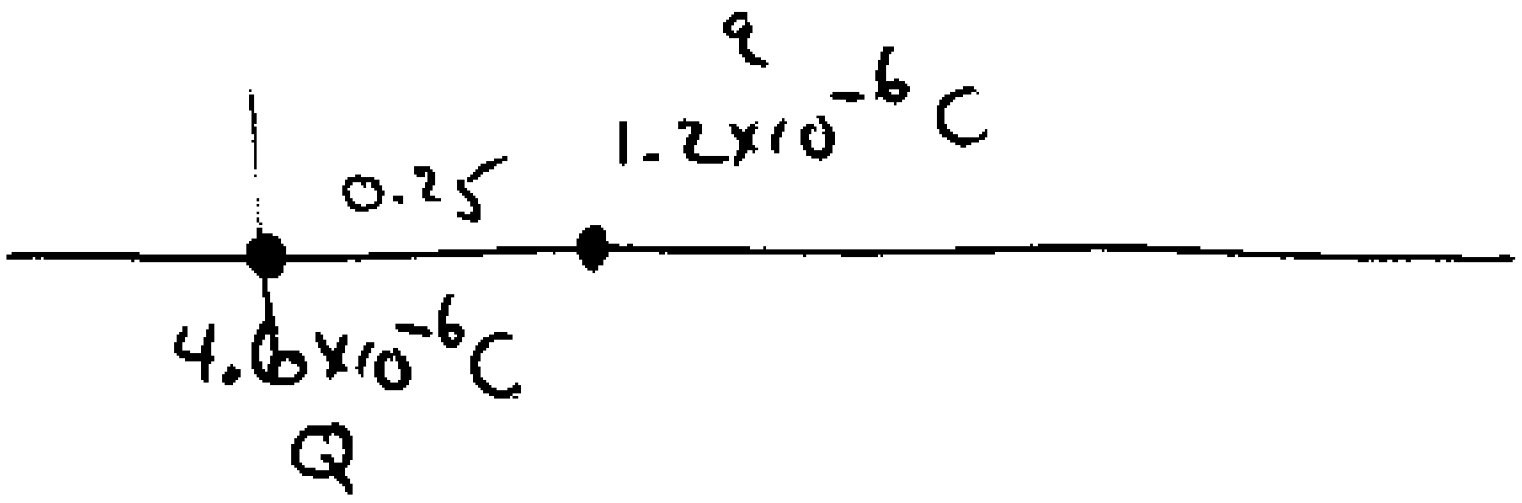
E&M Classwork 3 Solutions

1) $U = qV = \frac{qQ}{4\pi\epsilon_0}\frac{1}{r}$



so a) for q at 0.25m

$$U = \frac{1.2\times10^{-6}\ 4.6\times10^{-6}}{0.25}\ 8.99\times10^{9} = \underline{0.198\ J}$$

b) If release charge, it lose potential & gains kinetic energy

$$U_{initial} + \cancel{\tfrac{1}{2}mv^2_{initial}}_{\,0} = U_{final} + \tfrac{1}{2}mv^2_{final}$$

so $v_{final} = \left[\frac{2}{m}\left(0.198 - U_{final}\right)\right]^{1/2}$

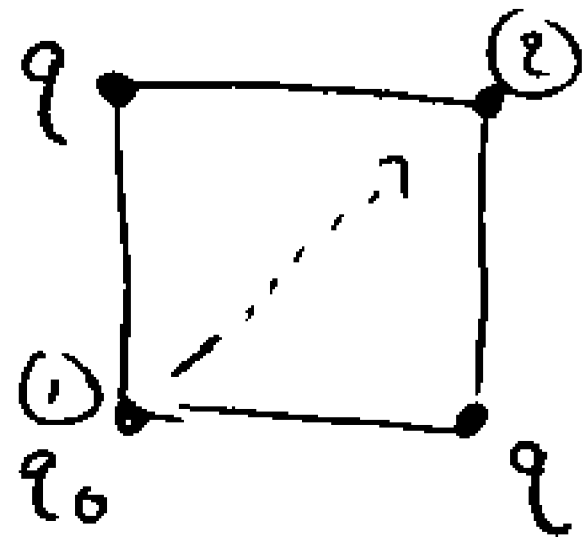
i) at $x = 0.5$, this is twice intiel pos $\therefore U_{final} = \frac{0.198}{2}$ and

$$v_{final} = \left[\frac{2}{2.8\times10^{-4}}\left(0.198 - \frac{0.198}{2}\right)\right]^{\frac{1}{2}} = 26.6\ m/s$$

ii) Sim at $x = 5$, $U_{final} = \frac{0.198}{20}$ and

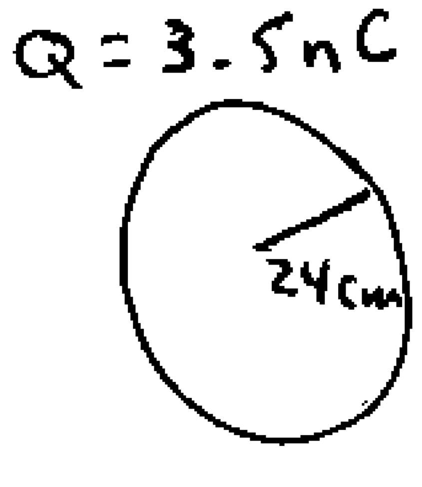
$$v_{final} = \left[\frac{2}{2.8\times10^{-4}}\left(0.198 - \frac{0.198}{20}\right)\right]^{\frac{1}{2}} = 36.7\ m/s$$

2) Work done by $\underset{\sim}{E}$ is just the difference in potential between start & end. But ① and ② are same distance from both chgs q, hence potential is same, hence work is <u>zero</u>



3) a) Outside potential is same as that of a pt charge, so $V(0.48m) = \frac{Q}{4\pi\epsilon_0}\frac{1}{0.48m}$

ie $V = 8.99\times10^{9}\ 3.5\times10^{-9}\ \frac{1}{0.48} = 65.6V$



b) at $r = 0.24m$ V is twice this, ie $V(0.24m) = 131\ V$

c) Inside $\underset{\sim}{E} = \underset{\sim}{0} \Rightarrow V = const = V(0.24cm) = 131V$

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(4) $$V = \frac{\mathbf{p}\cdot\mathbf{r}}{4\pi\epsilon_0 r^3} = \frac{q d z}{4\pi\epsilon_0 r^3}$$

recall $\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$|\mathbf{r}| = (x^2+y^2+z^2)^{1/2}$

The electric field is $$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$$

Now $$\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\frac{\partial}{\partial x}\left(\frac{qdz}{r^3}\right) = \frac{1}{4\pi\epsilon_0}\left[\frac{r^3\frac{\partial}{\partial x}(qdz) - qdz\frac{\partial r^3}{\partial x}}{r^6}\right]$$

and $$\frac{\partial r^3}{\partial x} = 3r^2\frac{\partial r}{\partial x} = 3r^2\tfrac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}(2x) = 3r^2\frac{x}{r} = 3rx$$

Thus $$\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\left[\frac{0 - qdz\,3rx}{r^6}\right] = -\frac{1}{4\pi\epsilon_0}\frac{(\mathbf{p}\cdot\mathbf{r})3x}{r^5}$$

sim $$\frac{\partial V}{\partial y} = -\frac{1}{4\pi\epsilon_0}\frac{(\mathbf{p}\cdot\mathbf{r})3y}{r^5}$$

while $$\frac{\partial V}{\partial z} = +\frac{1}{4\pi\epsilon_0}\left[\frac{r^3\frac{\partial}{\partial z}(qdz) - qdz\frac{\partial r^3}{\partial z}}{r^6}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{qd}{r^3} - \frac{3(\mathbf{p}\cdot\mathbf{r})z}{r^5}\right]$$

Putting these into $\mathbf{E} = -\nabla V$ gives

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\mathbf{p}\cdot\mathbf{r})x\hat{x}}{r^5} + \frac{3(\mathbf{p}\cdot\mathbf{r})y\hat{y}}{r^5} + \left(\frac{3(\mathbf{p}\cdot\mathbf{r})z\hat{z}}{r^5} - \frac{qd\hat{z}}{r^3}\right)\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3(\mathbf{p}\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{p}}{r^3}\right]$$

since $x\hat{x} + y\hat{y} + z\hat{z} = \mathbf{r}$ and $\mathbf{p} \equiv qd\hat{z}$