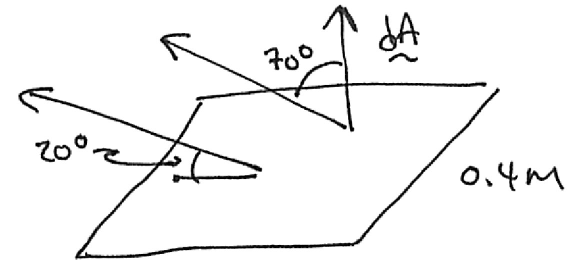
Electricity & Magnetism Classwork 2 Solutions

① $\Phi_E = \int \underset{\sim}{E} \cdot d\underset{\sim}{A}$



$= \int E\,dA \cos 70°$

Since $\underset{\sim}{E}$ uniform $\Phi_E = EA\cos 70°$

ie $\Phi_E = 75\frac{N}{C}\, 0.4m \times 0.6m \times \cos 70° = 6.16\ Nm^2/C$

② a) If surface encloses $-3.6\mu C$ then by Gauss' Law $\Phi_E = \frac{Q_{encl}}{\epsilon_0}$

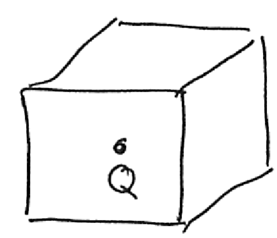
ie $\Phi_E = \frac{-3.6\times10^{-6}C}{8.85\times10^{-12}C/Nm^2} = -4.07\times10^5\ Nm^2/C$

b) Sim. $\Phi_E = 780\frac{Nm^2}{C} \Rightarrow Q_{encl} = \Phi_E\epsilon_0 = 780\times8.85\times10^{-12}$

$= 6.90\times10^{-9}C$

c) No. Gauss's Law says $\Phi_E = Q_{encl}/\epsilon_0$ regardless of where the charge is located as long as it is inside the surface

③ a) As Q is at centre of cube, by symmetry Φ_E through each face is the same. But by Gauss's Law, net Φ_E out of cube $= Q_{encl}/\epsilon_0$



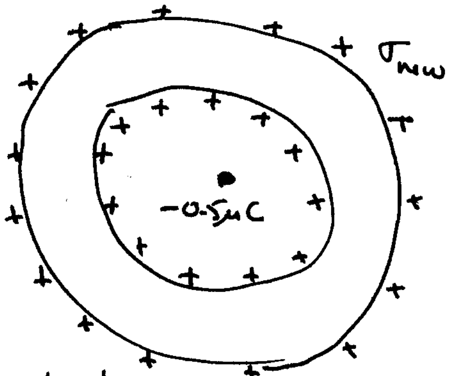
Hence Φ_E (one face) $= \frac{1}{6}\frac{Q_{encl}}{\epsilon_0} = \frac{1}{6}\frac{9.6\times10^{-6}C}{8.85\times10^{-12}C^2/Nm^2}$

$= 1.81\times10^5\ Nm^2/C$

b) Changing size of cube has no effect on Φ_E (note that size didn't enter into calculation for (a).)

E&M Classwork 2 2008 <u>solution to Q4</u>

(4) Y&F 22.23



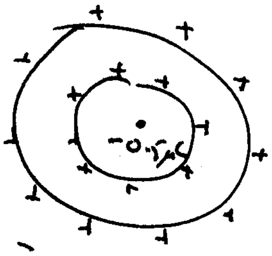
Although it's not obvious from the question, let's assume that the charge is placed at the <u>centre</u> of the cavity, so that spherical symmetry is preserved. This is needed for the answer! Since $\underset{\sim}{E} = \underset{\sim}{0}$ within the conductor, there must be a charge $+0.5\mu C$ on the inner surface.

a) That leaves $\left[6.37\mu\frac{C}{m^2} \times 4\pi(0.25)^2 - 0.5\mu C\right]$ on the outside, or a new surface density of

$$\sigma_{new} = 6.37\frac{\mu C}{m^2} - \frac{0.5\mu C}{4\pi(0.25)^2} = (6.37 - 0.637)\frac{\mu C}{m^2}$$

$$= 5.73\,\mu C/m^2$$

b) Take a Gaussian sphere just outside the shell. $\underset{\sim}{E} = E(r)\hat{r}$ by symmetry, so



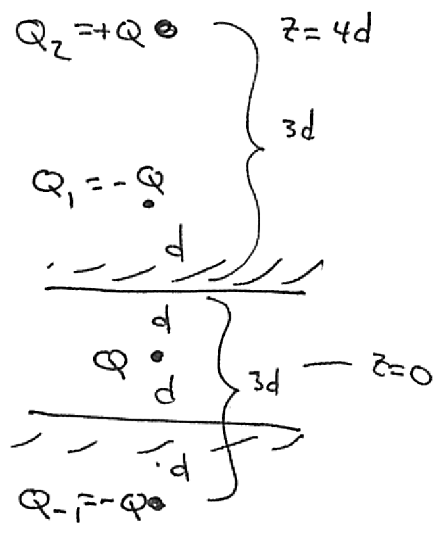
$$E(0.25m)\, 4\pi(0.25)^2 m^2 = \frac{Q_{encl}}{\epsilon_0} = \frac{5.73\mu C \times 4\pi(0.25)^2 \cancel{m^2}}{\cancel{m^2}\,\epsilon_0}$$

[The net chg on system is all on outside, hence $= \sigma_{new} \times 4\pi(0.25)^2$]

ie
$$E = \frac{5.73\times10^{-6}\,C/m^2}{8.85\times10^{-12}\,C^2/Nm^2} = 6.47\times10^5\,\frac{N}{C} \qquad \left[\frac{N}{C} = \frac{V}{m}\right]$$

c) By Gauss's Law
$$\Phi_E = \frac{Q_{encl}}{\epsilon_0} = \frac{-0.5\times10^{-6}C}{8.85\times10^{-12}\,C^2/Nm^2} = -5.65\times10^4\,\frac{Nm^2}{C}$$

(5) Q will give rise to images Q_1 and Q_{-1} at $z = \pm 2d$ of magnitude $-Q$



But then Q_{-1} will have image Q_2 at $z=4d$

sim Q_{+1} " " " Q_{-2} " $z=-4d$

Each of these gives another image ⇒ ∞ number (just like ∞ number of images if you stand between 2 mirrors). In general then we'll have $Q_n = (-1)^n Q$ at $z_n = 2nd$ for $n = 0, \pm 1, \pm 2, \dots$

Each will give $E_n = \dfrac{Q_n/4\pi\epsilon_0}{|z-2nd|^2} \dfrac{z-2nd}{|z-2nd|} \hat{z}$

Put $z = -d$. Then

$$E_0 = \frac{Q}{4\pi\epsilon_0} \frac{1}{(-d)^2} \frac{-d}{|-d|} \hat{z} = -\frac{Q}{4\pi\epsilon_0} \frac{1}{d^2} \hat{z}$$

$$E_1 = \frac{-Q}{4\pi\epsilon_0} \frac{1}{(-3d)^2} \frac{-3d}{|-3d|} = +\frac{Q}{4\pi\epsilon_0} \frac{1}{d^2} \frac{1}{3^2} \hat{z}$$

$$E_{-1} = \frac{-Q}{4\pi\epsilon_0} \frac{1}{(+d)^2} \frac{+d}{|+d|} \hat{z} = -\frac{Q}{4\pi\epsilon_0} \frac{1}{d^2} \hat{z}$$

$$E_2 = \frac{+Q}{4\pi\epsilon_0} \frac{1}{(-5d)^2} \frac{-5d}{|-5d|} = -\frac{Q}{4\pi\epsilon_0} \frac{1}{d^2} \frac{1}{5^2} \hat{z}$$

$$E_{-2} = \frac{Q}{4\pi\epsilon_0} \frac{1}{(3d)^2} \frac{3d}{|3d|} = +\frac{Q}{4\pi\epsilon_0} \frac{1}{d^2} \frac{1}{3^2} \hat{z}$$

etc. so

$$\underset{\sim}{E}(-d) = \sum_{n=-\infty}^{\infty} E_n = \frac{2Q}{4\pi\epsilon_0} \frac{1}{d^2} \left[-1 + \frac{1}{3^2} - \frac{1}{5^2} + \frac{1}{7^2} - \dots \right] \hat{z}$$