7.6 Magnetic Materials

7.6.1 Motivation

Many materials respond to the presence of a magnetic field. Some, for example, are strongly attracted to a bar magnet; others only weakly if at all. Some can amplify significantly the magnetic field. This is why motors and electromagnets have wires that are wound around an iron core instead of air. Some materials can be "magnetised" so that they retain a permananet magnetisation even after the applied field is removed.

Yet others result in a decrease of the magnetic field strength, just as dielectrics reduce the electric field.

In our course, we only need to introduce a few concepts in order to be able to use magnetic materials. The purpose of this handout is to lay out in a bit more detail the different kinds of magnetic materials and their physics. Young & Freedman cover this subject in section 28.8 of the 12th edition.

7.6.2 Magnetic Materials: Basics

There are three main categories of magnetic materials:

Paramagnetic materials are made up of atoms that possess a net magnetic dipole moment. In the presence of an applied magnetic field, the torque $\boldsymbol{\mu}_m \times \mathbf{B}$ tends to make the dipoles align with the field. Since a magnetic dipole produces a magnetic field that points in the same direction as $\boldsymbol{\mu}_m$, the applied field and that from the dipole add, so that the total field is larger than the applied one. The effect is quite weak, with the field enhanced by a factor $\sim 1.00001 - 1.003$. Paramagnetic materials include uranium, platinum, aluminium, and sodium.

Diamagnetic materials have no magnetic properties on their own (i.e., no intrinsic atomic magnetic moment), but their atomic configuration means that the electron orbits are modified in the presence of an applied magnetic field. Electrons circulating in a magnetic field are equivalent to a current loop with a magnetic moment directed in the *opposite* direction to the applied magnetic field (see Figure 1) and so they tend to *decrease* the total field. Again the effect is small, with the magnitude of the field reduced by a factor $\sim 0.99999 - 0.9999$. Diamagnetic materials include mercury, silver, carbon, lead, and copper.

Ferromagnetic materials have "magnetic domains" that don't move easily. But in the present of an external magnetic field, the domains that are aligned with that field grow at the expense of the others. The result is a strong enhancement in magnetic fields (factors 1000-100,000). Moreover, even when the applied field is taken away, these materials retain their magnetisation and thus form "permanent magnets." Ferromagnetic materials include iron (obviously!), nickel, cobalt, and many alloys of these materials.

7.6.3 Formulation

Paramagnetism and Diamagnetism

The mathematical key to magnetic materials is the magnetisation vector **M**, which is basically the total magnetic moment per unit volume. The magnetic field attributable to that magnetisation is simply $\mu_o\mathbf{M}$. It is easy to see that this has the right units, but less easy to verify that it gives the correct numerical value. Thus the total field is the externally applied one plus that due to **M**, i.e.,

$$\mathbf{B} = \mathbf{B}_o + \mu_o\mathbf{M} \tag{1}$$

Paramagnetic and diamagnetic materials respond more or less linearly to an applied field, so that $\mathbf{M} \propto \mathbf{B}_o$, so we can write:

$$\begin{aligned} \mathbf{B} &= (1+\chi_m)\,\mathbf{B}_o & (2)\\ &= K_m\mathbf{B}_o & (3) \end{aligned}$$

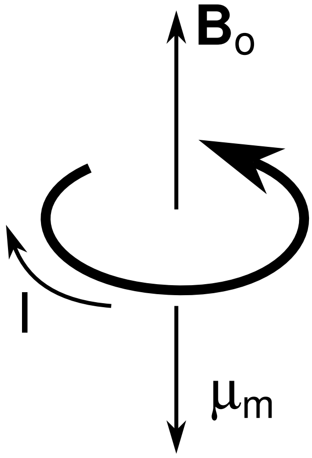

Figure 1: An electron circulating under the influence of the Lorentz force in a magnetic field $\mathbf{B}_o$. The orbit can be thought of as a current loop carrying a positive current in the opposite sense. That current loop has a magnetic moment $\boldsymbol{\mu}_m$ which is directed in the opposite direction to $\mathbf{B}_o$. The magnetic field due to the current loop is thus also opposite to $\mathbf{B}_o$ within the loop, and thus the total magnetic field is reduced from $\mathbf{B}_o$.

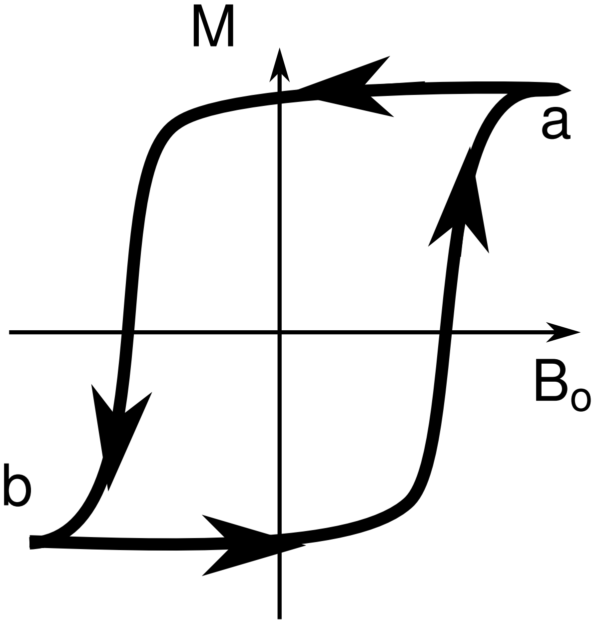

Figure 2: Hysteresis in the magnetisation of a ferromagnetic material. From its saturated values at points (a) and (b), the material retains its magnetisation until the external field B_o has reversed in sign and reached an appreciable magnitude.

where we have introduced the **magnetic susceptibility** χ_m and the **relative permeability** K_m. For paramagnetic materials, χ_m is positive so that $K_m > 1$ while for diamagnetic materials χ_m is negative and $K_m < 1$. However, as noted above these effects are relatively weak, so that K_m is only marginally different from unity in both cases.

Ferromagnetism

Ferromagnetic materials are often described using the same language of relative permeabilities and such, so that mathematically they are treated in the same way. However, they are *not* typically very linear. That is, the magnetisation $\mathbf{M}$ is *not* directly proportional to the applied field $\mathbf{B}_o$. Indeed, their magnetic domains exhibit a phenomenon known as **hysteresis**, which is illustrated in Figure 2.

From an initial state labelled (a) in the figure, decreasing the applied magnetic field initially has relatively little effect, and the magnetisation decreases only slightly until the applied field has not only reversed direction but also has a significant magnitude. Eventually the magnetisation does reverse and eventually reaches the point labelled (b).

Notice that the magnetisation is limited; increasing B_o to values to the right of (a) has relatively little further effect on M. The material **saturates** and no further magnetisation is possible.

Different materials have different hysteresis curves. Some are relatively narrow (in B_o), so that they respond more readily to smaller external fields and smaller variations in them. Others, for which the hysteresis loop encloses a large volume, are much more stubborn. The area of the loop is related to the amount of energy that is dissipated in the process: the larger the loop the more heat is generated.

Notice also that most of the magnetisation is retained even when the applied magnetic field is removed (i.e. $B_o = 0$). Thus a ferromagnetic material becomes permanently magnetised. It should also be clear that ferromagnetic materials are hard to de-magnetise.

Despite this interesting, distinctly nonlinear behaviour of the magnetisation and hence the total magnetic field in the case of ferromagnetism, it is nonetheless common to describe them with the same mathematical language in terms of χ_m and K_m. In practive, only K_m is needed as it is $\gg 1$. But Figure 2 shows that K_m is a strong function of both B_o and of the history of the process.

7.6.4 Conclusion

The magnetic behaviour of materials can be quite interesting, varying from decreasing, albeit slightly, any applied field in the case of diamagnetic materials, through some small enhancement in the case of paramagnetic materials, to the large, nonlinear, and permanent enhancements found in ferromagnetic materials. For everything we shall need in the present course, it is sufficient to treat the response mathematically using Equation 3, i.e.,

$$\mathbf{B} = K_m \mathbf{B}_o$$

though for ferromagnetic materials the history of the process is actually important. Finally, you will often see the magnetic properties embodied in an expression of the magnetic permeability

$$\mu \equiv K_m \mu_o$$

and in most, if not all, cases you can recover the correct result in the presence of a material by replacing the vacuum permeability μ_o by μ.

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