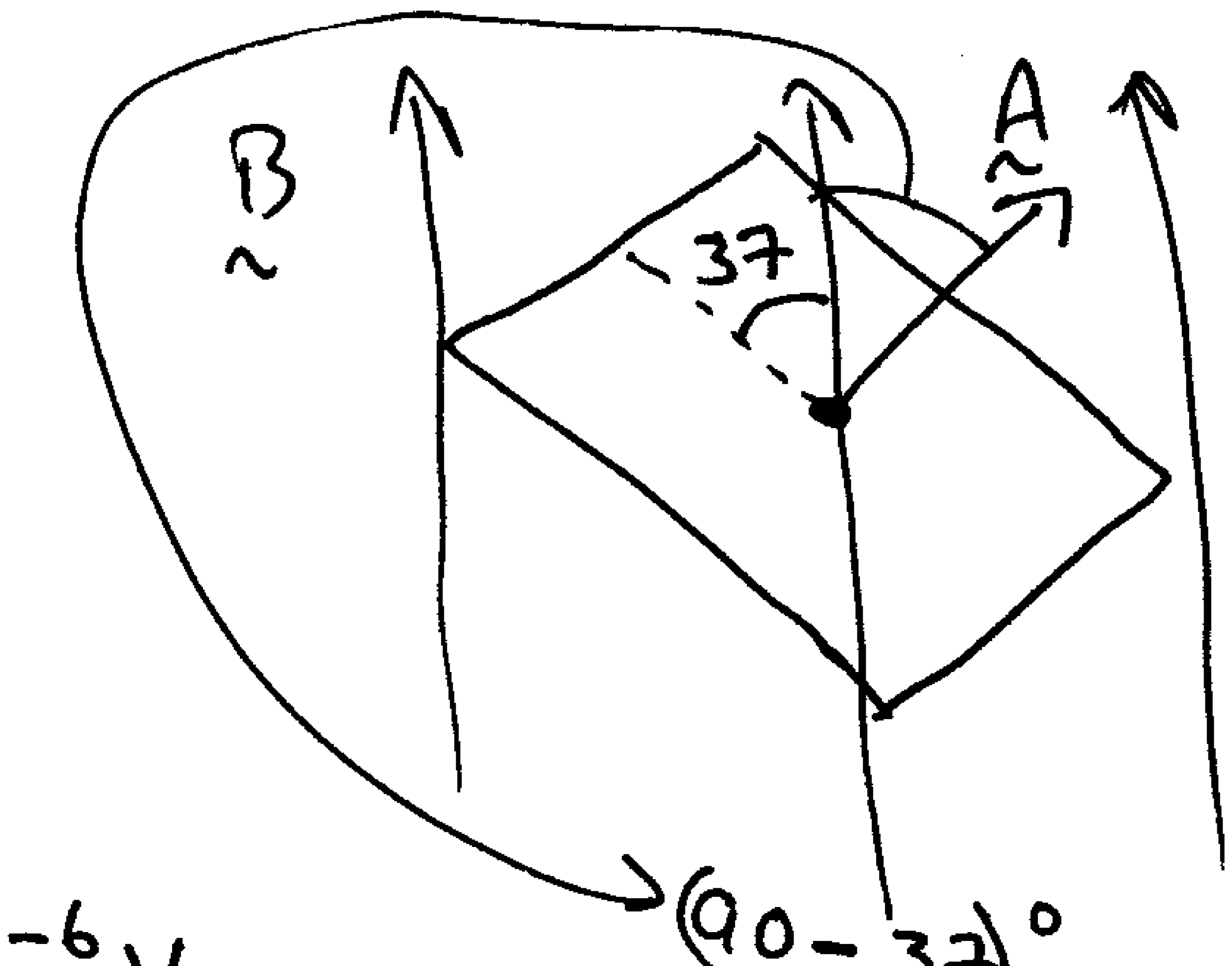
Elec & Magnetism Prob. Sheet 8 Solns

① $\Phi_B = \iint \underset{\sim}{B} \cdot d\underset{\sim}{A} = BA\cos 53^\circ$

$A = (0.1)^2 m^2$

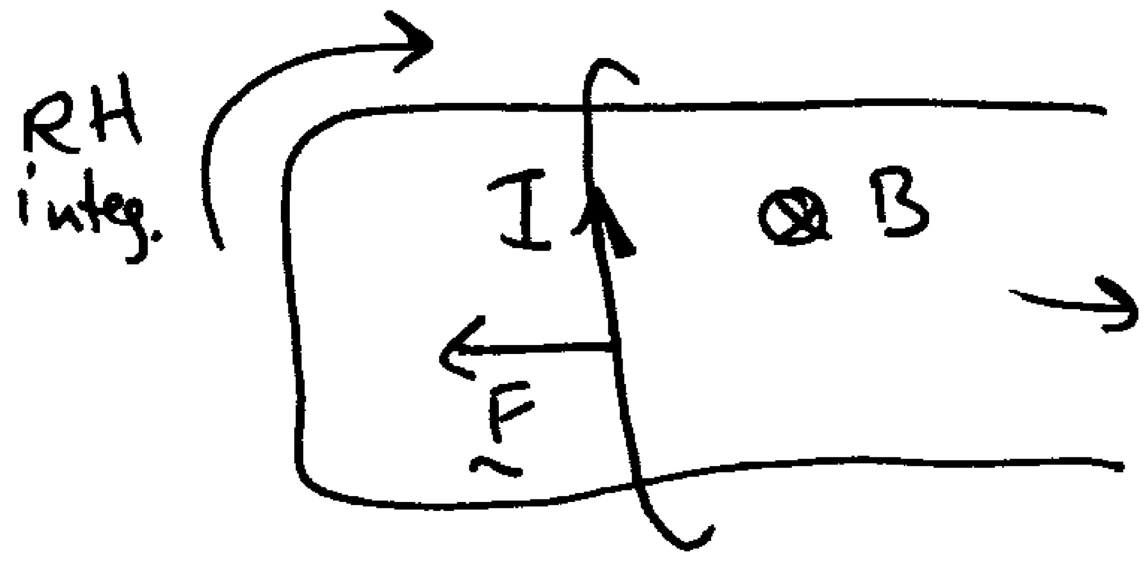


$$\mathcal{E} = -\frac{d\Phi_B}{dt} = A\cos 53^\circ \frac{dB}{dt}$$

since $B = 10^{-3}t + 0.1 \Rightarrow \frac{dB}{dt} = 10^{-3}\,T/s$

so $\mathcal{E} = (0.1)^2 \cos 53^\circ\, 10^{-3} = \underline{\underline{6.02 \times 10^{-6}\,V}}$

② a) Need to find $I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)$



ie $I = -\frac{1}{R} BLv$ since $\frac{dA}{dt} = +Lv$

Then $\underset{\sim}{F} = I\underset{\sim}{L} \times \underset{\sim}{B} = \left(-\frac{1}{R}BLv\right) LB\hat{x}$

Thus $\underline{F = B^2L^2v/R}$

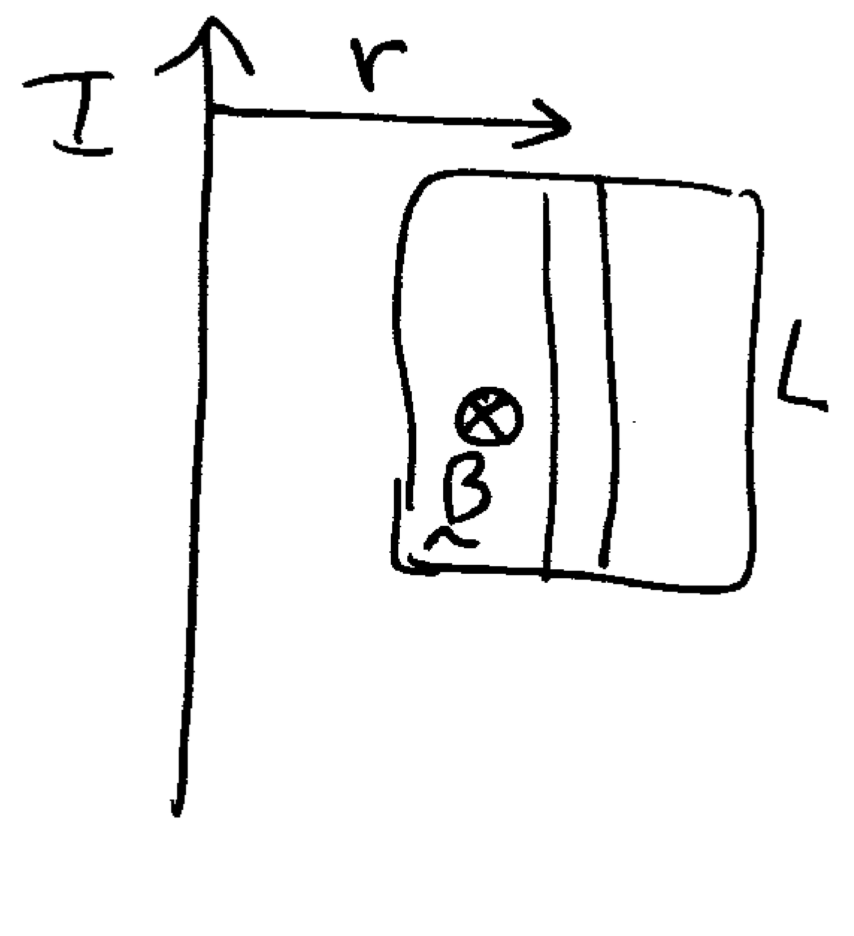
[NB $\underset{\sim}{I}$ is negative wrt RH integration, so actual I has direction as shown]

b) Eqn of motion $m\frac{dv}{dt} = F = -B^2L^2v/R$

$\Rightarrow \frac{1}{v}\frac{dv}{dt} = -\frac{B^2L^2}{Rm}$ Integ $0 \to t \Rightarrow v(t) = v_0 \exp\left(-\frac{B^2L^2}{Rm}t\right)$

Finally $x = \int_0^\infty v(t)\,dt = \int_0^\infty v_0 \exp\left(-\frac{B^2L^2}{Rm}t\right)dt = \underline{\frac{mv_0R}{B^2L^2}}$

③ a) $\underset{\sim}{B}$ due to long straight wire is around wire in RH sense with magnitude $2\pi r B(r) = \mu_0 i$



so $\underset{\sim}{B} = \frac{\mu_0 i}{2\pi}\frac{1}{r}$ (into page)

b) So $d\Phi_B = \frac{\mu_0 i}{2\pi}\frac{1}{r}L\,dr$ ($\equiv \underset{\sim}{B}\cdot d\underset{\sim}{A}$ with $d\underset{\sim}{A}$ into page)

c) So $\Phi_B = \int_{r=a}^{b} \frac{\mu_0 i}{2\pi}\frac{L}{r}dr = \frac{\mu_0 i L}{2\pi}\ln(b/a)$

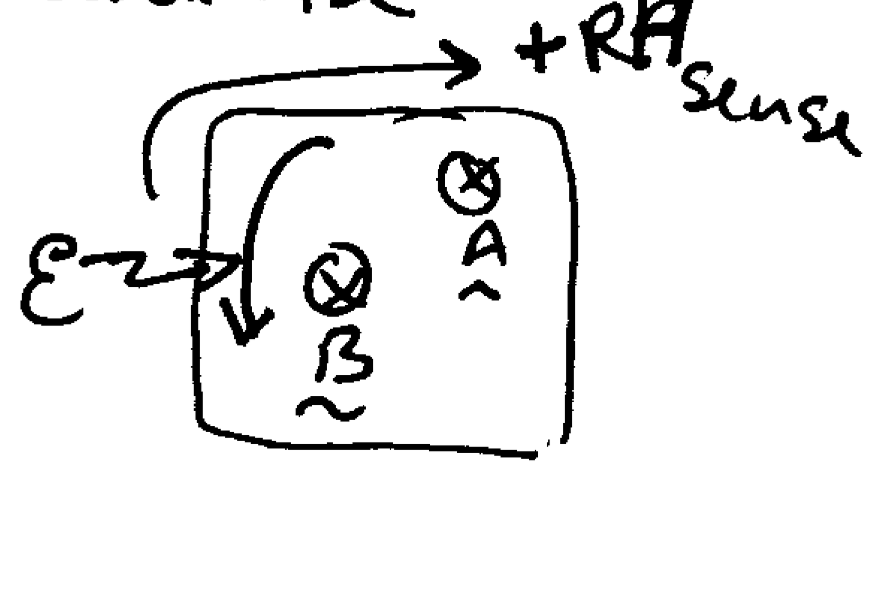
d) so $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 L}{2\pi}\ln(b/a)\frac{di}{dt}$

For $\frac{di}{dt} > 0$, $\mathcal{E}$ is negative so would drive I counterclockwise

e) Put in given numbers

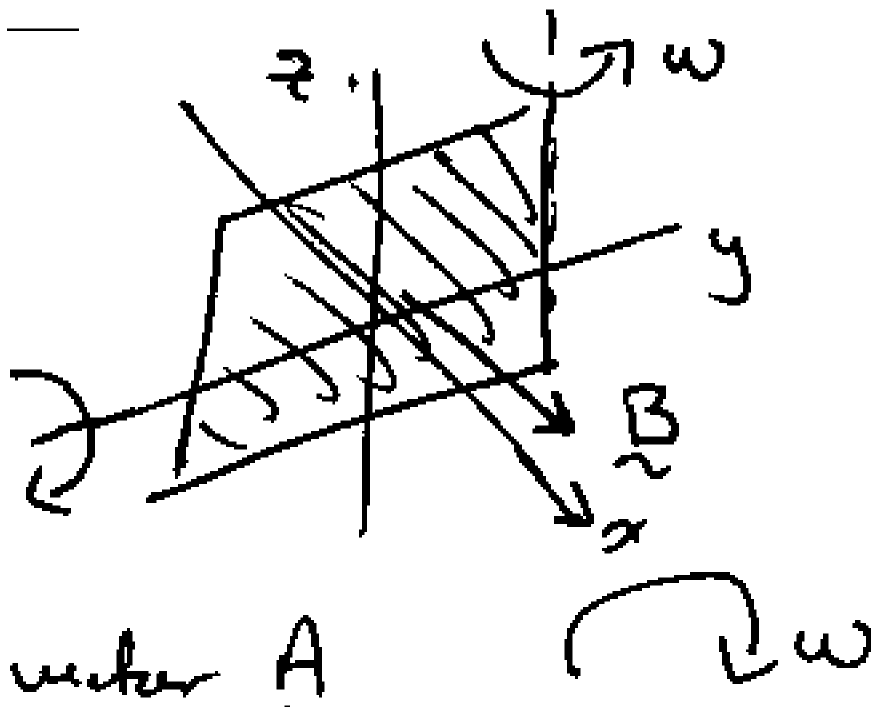
$$\mathcal{E} = -\frac{4\pi \times 10^{-7}\, 0.24}{2\pi}\ln\left(\frac{36}{12}\right) 9.6$$

$= \underline{\underline{5 \times 10^{-7}\,V}}$



Electricity & Magnetism Solutions

(4) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \underset{\sim}{B}\cdot d\underset{\sim}{A} = -\frac{d}{dt}[BA\cos\phi]$



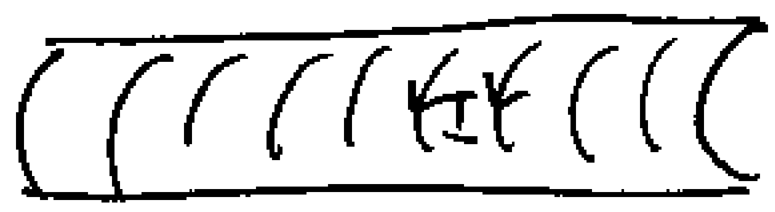
$= -BA\frac{d}{dt}(\cos\phi)$ where ϕ is angle between $\underset{\sim}{B}$ and Area vector $\hat{A}$

For rotation about $\hat{y}$ (case a) and edge $\parallel \hat{z}$ (case c) $\phi = \omega t$ [case (c) also involves a <u>translation</u> of the centre of the area, but that doesn't affect Φ_B, only the angle matters]. For rotation about $\hat{x}$ (case b) $\phi = 0$. Hence

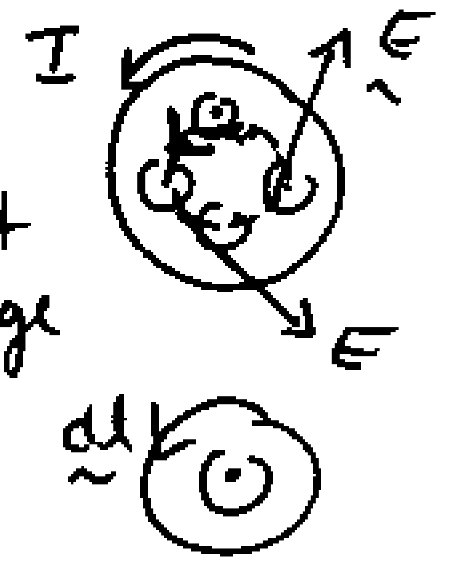
(a, c) $\mathcal{E} = +BA\omega\sin\omega t$ so $\mathcal{E}_{max} = BA\omega = 0.45\times0.6\times35$
$= 9.45\ V$

b) $\mathcal{E} = 0$

(5) $\oint \underset{\sim}{E}\cdot d\underset{\sim}{l} = -\frac{d\Phi_B}{dt}$ By symmetry, $\underset{\sim}{E}$ is tangent to circles



so $\oint \underset{\sim}{E}\cdot d\underset{\sim}{l} = E\,2\pi r = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(\underset{\sim}{B}\cdot d\underset{\sim}{A})$



ie $E\,2\pi r = -\pi r^2\frac{dB}{dt} = -\pi r^2\frac{d}{dt}(\mu_0 n I)$

Thus $E = -\frac{r\mu_0 n}{2}\frac{dI}{dt}$

So, at centre, $r=0$ and hence $E=0$

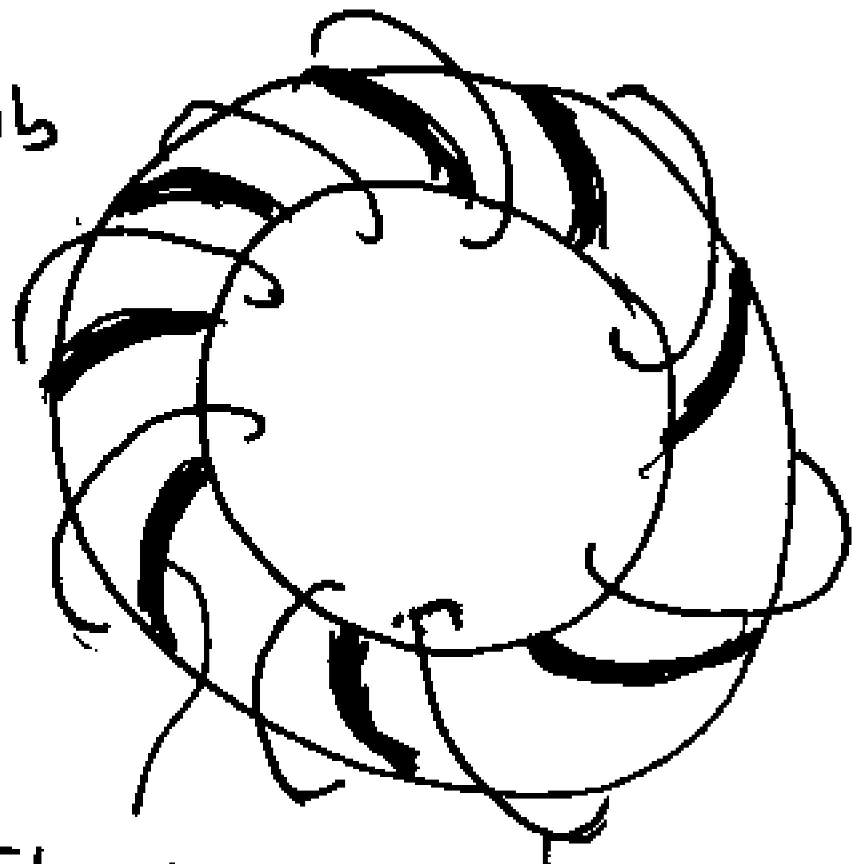
a) at $r = 0.5$cm $|\underset{\sim}{E}| = +\frac{0.5\times10^{-2}\,4\pi\times10^{-7}}{2}\,\frac{\times900}{60} = 1.70\times10^{-4}\ V/m$

b) at $r = 1.0$ cm E is twice that at 0.5, ie $E = 3.40\times10^{-4}\ V/m$

[Direction is in LH sense about $\underset{\sim}{B}$: not asked but useful]

Electricity & Magnetism Prob Sheet 8 solutions

(6) Given $i_1 = 6.52\,A$ & $\Phi_{B2} = 0.032\,Wb$



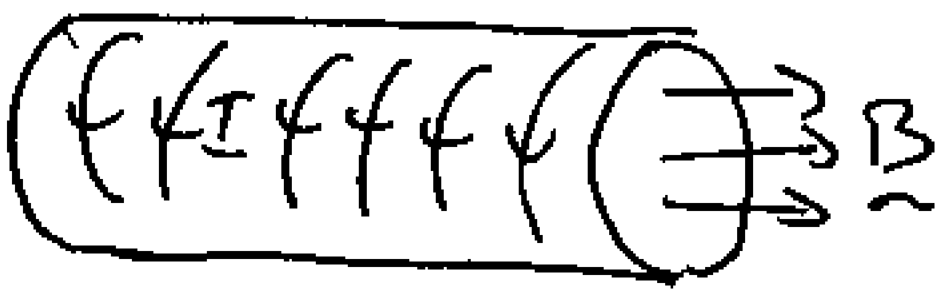
a) $$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{400 \times 0.032}{6.52}$$

$$= \underline{1.96\,H}$$

b) $$\Phi_{B1} \text{ (each turn)} = \frac{M i_2}{N_1} = \frac{1.96 \times 2.54}{700}$$

$$= 0.0071\,Wb$$

(7) $$L = \frac{N \Phi_B}{i} = \frac{NBA}{i}$$



But for long solenoid $B = \mu_0 n I = \mu_0 \frac{N}{l} I$ so

$$L = \frac{N \mu_0 \frac{N}{l} i A}{i} = \frac{N^2 A}{l} \mu_0$$