First Year Electricity & Magnetism **Problem Sheet 8** **10 March 2008**

Magnetic Induction

Numbers in italics refer to the Exercise number in the 12th Edition of Young & Freedman's "University Physics". Answers to odd-numbered questions are at the back of the book. There are a lot of problems here, to keep you out of trouble during the vacation!

1. A square copper loop, 10.0 cm on a side, is located in a region of changing magnetic field. The direction of the magnetic field makes an angle of 37° with the plane of the loop. The time-changing field has the following time dependence: $B(t) = 0.10\,\mathrm{T} + (1.00 \times 10^{-3}\,\mathrm{T/s})t$. Find the induced emf in the cooper loop for times $t > 0$.

2. [*29.65*] A rectangular loop with width L and a slide wire with mass m are as shown in the Figure. A uniform magnetic field **B** is directed perpendicular to the plane of the loop into the plane of the figure. The slide wire is given an initial speed of v_0 and then released. There is no friction between the slide wire and the loop, and the resistance of the loop is negligible in comparison to the resistance R of the slide wire. a) Obtain an expression for F, the magnitude of the force exerted on the wire while it is moving at speed v. b) Show that the distance x that the wire moves before coming to rest is $x = mv_0R/L^2B^2$. [*Hint: Find the current I that flows around the end of the loop and along the sliding wire. Thus find F. Integrate the equation of motion to show that* $\ln(v/v_0) = -(B^2L^2/Rm)t$.]

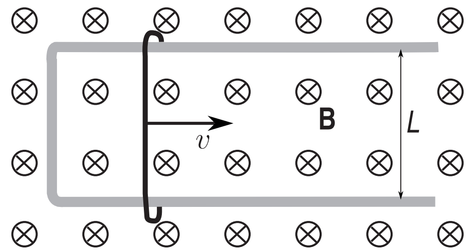

Figure from Q29.65.

3. [*29.7*] **BONUS** The current shown in the long [infinite], straight wire is upward and is increasing steadily at a rate di/dt. a) At an instant when the current is i, what are the magnitude and direction of the field **B** at a distance r to the right of the wire? b) What is the flux $d\Phi_B$ through the narrow, shaded strip? c) What is the total flux through the loop? d) What is the induced emf in the loop? e) Evaluate the numerical value of the induced emf if a = 12.0 cm, b = 36.0 cm, L = 24.0 cm, and di/dt = 9.60 A/s. [*The loop is NOT necessarily square.*]

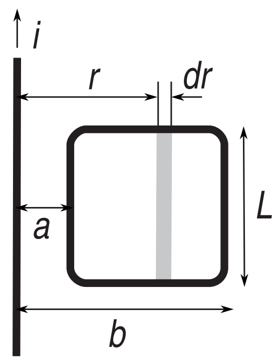

Figure from Q29.7.

4. [*29.50*] Suppose the loop in the Figure shown is a) rotated about the y-axis; b) rotated about the x-axis; c) rotated about an edge parallel to the z-axis. What is the maximum induced emf in each case if the area A of the loop is 600 cm, ω =35.0 rad/s, and B = 0.450 T? [**B** is uniform.]

5. [*29.27*] A long, thin solenoid has 900 turns per metre and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 60.0 A/s. What is the magnitude of the induced electric field at a point near the centre of the solenoid and a) 0.500 cm from the axis of the solenoid? b) 1.00 cm from the axis of the solenoid.

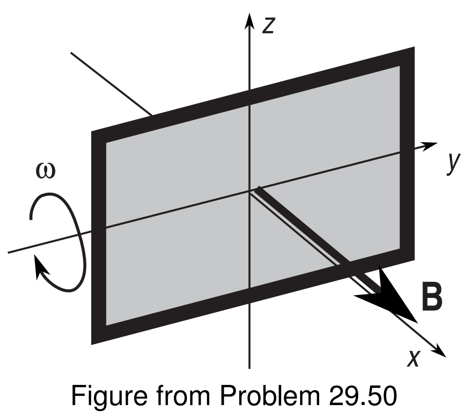

Figure from Problem 29.50

6. [*30.5*] Two toroidal solenoids are wound around the same form so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb. a) What is the mutual inductance of the pair of solenoids? b) When the current in solenoid 2 is 2.54 A, what is the average flux through each turn of solenoid 1?

7. [*30.11*] A long, straight solenoid has N turns, uniform cross-sectional area A, and length ℓ. Show that the inductance of this solenoid is given by the equation $L = \mu_o AN^2/\ell$. Assume that the magnetic field is uniform inside the solenoid and zero outside. (Your answer is approximate because B is actually smaller at the ends than at the centre. For this reason, your answer is actually an upper limit on the inductance.)

Some possibly useful information and formulae:

$$
\begin{aligned}
B &= \mu_0 nI \\
B &= \frac{\mu_0 NI}{2\pi r} \\
\mathcal{E} &= -\frac{d\Phi_B}{dt} \\
\oint \mathbf{E}\cdot\mathbf{d}\boldsymbol{\ell} &= -\frac{d\Phi_B}{dt}
\end{aligned}
\qquad
\begin{aligned}
\mathcal{E}_2 &= -M\frac{di_1}{dt} \\
\mathcal{E}_1 &= -M\frac{di_2}{dt} \\
M &= \frac{N_2\Phi_{B2}}{i_1} \\
&= \frac{N_1\Phi_{B1}}{i_2}
\end{aligned}
\qquad
\begin{aligned}
\mathcal{E} &= -L\frac{di}{dt} \\
L &= \frac{N\Phi_B}{i} \\
\mu_0 &= 4\pi\times 10^{-7}\,\mathrm{Tm/A}
\end{aligned}
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My best wishes for your break and your success next term.

Steve Schwartz 10 March 2008