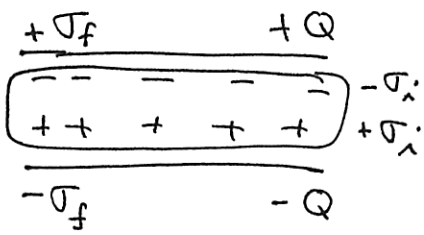
E&M Problem Sheet 4 Solutions

1) a) for $\parallel$ plates $|\underset{\sim}{E}| = \dfrac{|\sigma|}{\epsilon_0}$



Introduction of dielectric reduces E from 3.20×10^5 to 2.50×10^5 V/m

so σ_i is responsible for $(3.20 - 2.50) \times 10^5 = 0.7 \times 10^5$ V/m

Hence $\sigma_i = 0.7 \times 10^5 \times \epsilon_0 = 0.7 \times 10^5 \times 8.85 \times 10^{-12} = \underline{6.20 \times 10^{-7}\ C/m^2}$

b) Dielectric reduces $E_0 \Leftrightarrow \sigma_{free}$ by ϵ_r, so $E = E_0/\epsilon_r$

$$\epsilon_r = \frac{E_0}{E} = \frac{3.20 \times 10^5}{2.50 \times 10^5} = \underline{1.28}$$

c) $\underset{\sim}{D} = \epsilon_r \epsilon_0 \underset{\sim}{E} = \epsilon_0 \underset{\sim}{E}_0 \Rightarrow |\underset{\sim}{D}| = \epsilon_0 \underset{\sim}{E}_0 = 8.85 \times 10^{-12}\ 3.2 \times 10^5$

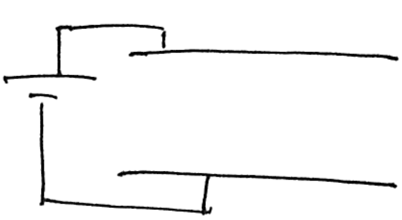
ie $|\underset{\sim}{D}| = \underline{2.83 \times 10^{-6}\ C/m^2}$

[These units are obvious from Gauss: $\oiint \underset{\sim}{D} \cdot d\underset{\sim}{A} = Q_{free}$; to get them from units of ϵ_0, etc. note force $\underset{\sim}{F} = q\underset{\sim}{E}$ shows that N = CV/m]

2) a) Before $C = 12.5\,\mu F$, so

$U_{before} = \frac{1}{2} CV^2 = \frac{1}{2} 12.5 \times 10^{-6} (24)^2$

$= 3.6 \times 10^{-3}$ J



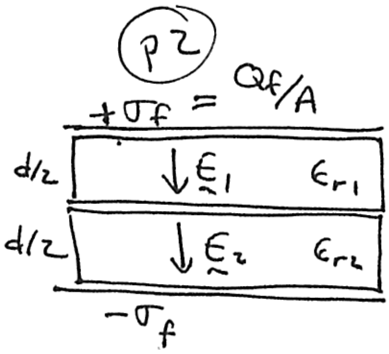
After, C has increased by ϵ_r (need more charge to get to same V)

so $U_{after} = \frac{1}{2}\ 3.75 \times 12.5 \times 10^{-6} (24)^2 = \underline{1.35 \times 10^{-2}\ J}$

b) so $U_{after} - U_{before} = (13.5 - 3.6) \times 10^{-3} = 9.9 \times 10^{-3}$ J

Thus energy increased.

E&M Prob. Sheet 4 Solutions



3) Action of dielectric is to reduce $|\underset{\sim}{E}|$ from its vacuum value by ϵ_r.

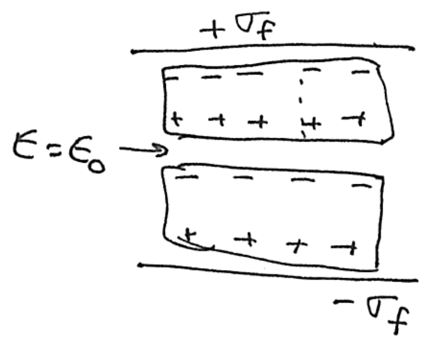
so $|\underset{\sim}{E}_1| = \dfrac{E_0}{\epsilon_{r1}}$, $|\underset{\sim}{E}_2| = \dfrac{E_0}{\epsilon_{r2}}$

with $E_0 = \sigma_f/\epsilon_0 = Q_f/\epsilon_0 A$. Now find total voltage drop across slabs:

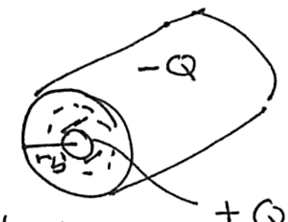
$$V = \frac{d}{2}E_1 + \frac{d}{2}E_2 = \frac{d}{2}E_0\left(\frac{1}{\epsilon_{r1}} + \frac{1}{\epsilon_{r2}}\right) = \frac{d}{2}\frac{Q_f}{\epsilon_0 A}\left(\frac{\epsilon_{r1}+\epsilon_{r2}}{\epsilon_{r1}\epsilon_{r2}}\right)$$

Now $C = \dfrac{Q_f}{V} = \dfrac{2\epsilon_0 A}{d}\left(\dfrac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1}+\epsilon_{r2}}\right)$ as required.

[If you're worried about whether one dielectric influences $\underset{\sim}{E}$ inside the other, imagine an infinitesimally small gap between them. In that gap $\underset{\sim}{E} = \underset{\sim}{E}_0$]



4) put $+Q$ on inner cylinder, $-Q$ on outer.
Take Gaussian cylinder at r & apply Gauss:



$$\oint\!\!\oint \underset{\sim}{D}\cdot d\underset{\sim}{A} = Q = D\,2\pi r L = \epsilon_r\epsilon_0 E_r 2\pi r L$$

since by symmetry $\underset{\sim}{D} = D(r)\hat{r}$ [could use small length $l \ll L$ to be sure end effects negligible]. So

$$E_r = \frac{Q}{\epsilon_r\epsilon_0 2\pi L}\frac{1}{r} \Rightarrow V_{ab} = \frac{Q}{\epsilon_r\epsilon_0 2\pi L}\int_{r_a}^{r_b}\frac{1}{r}\,dr \quad (*)$$

ie $V_{ab} = \dfrac{Q}{\epsilon_r\epsilon_0 2\pi L}\ln(r_b/r_a)$

Then $C \equiv \dfrac{Q}{V_{ab}} = \dfrac{2\pi\epsilon_r\epsilon_0 L}{\ln(r_b/r_a)}$ as req'd

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(*) [We have been a little sloppy with minus signs here, but we only need $|V_{ab}|$ and $|Q|$ etc. so no harm]
$V_a - V_b \equiv -\int_b^a E_r\,dr$