First Year Electricity & Magnetism **Problem Sheet 4** **13 February 2008**

Capacitors and Dielectrics

Numbers in italics refer to the Exercise number in the 12th Edition of Young & Freedman's "University Physics". Answers to odd-numbered questions are at the back of the book.

1. [*24.39*] Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the [magnitude of the] electric field is $E = 3.20 \times 10^5$ V/m. When the space is filled with dielectric, the [magnitude of the] electric field is $E = 2.50 \times 10^5$ V/m. (a) What is the charge density on each surface of the dielectric? (b) What is the dielectric constant? (c) [not in book] What is the magnitude of the displacement vector **D** within the dielectric?

2. [*24.47*] A 12.5 μF capacitor is connected to a power supply that keeps a constant potential difference of 24.0 V across the plates. A piece of material having a dielectric constant [ϵ_r] of 3.75 is placed between the plates, completely filling the space between them. (a) How much energy is stored in the capacitor before and after the dielectric is inserted? (b) By how much did the energy change during the insertion? Did it increase or decrease?

3. [*24.71*] A parallel-plate capacitor [with plates of area A] has the space between the plates filled with two slabs of dielectric [each of which is also of area A]; one with constant ϵ_{r1} and one with constant ϵ_{r2}. Each slab has thickness $d/2$, where d is the plate separation. Show that the capacitance is

$$C = \frac{2\epsilon_o A}{d}\left(\frac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}}\right)$$

4. A capacitor is formed from two conducting concentric cylinders of radii r_a and $r_b > r_a$ and length L. The space inbetween the two cylinders is filled with a dielectric material with dielectric constant ϵ_r. Show that the capacitance of this system is given by

$$C = \frac{2\pi\epsilon_r\epsilon_0 L}{\ln(r_b/r_a)}$$

Some possibly useful constants and formulae:

$$\begin{aligned}
E &= \frac{\sigma}{\epsilon_o} \\
\oint\!\!\oint \mathbf{D}\cdot\mathbf{dA} &= Q_{free} \\
C &= Q/V \\
\mathbf{D} &= \epsilon_r\epsilon_o\mathbf{E} \\
u_E &= \frac{1}{2}\epsilon_o E^2 \\
u_E &= \frac{1}{2}\epsilon_r\epsilon_o E^2 \\
U &= \frac{1}{2}CV^2 = \frac{1}{2}QV \\
\epsilon_o &= 8.85 \times 10^{-12}\, C^2/Nm^2
\end{aligned}$$

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