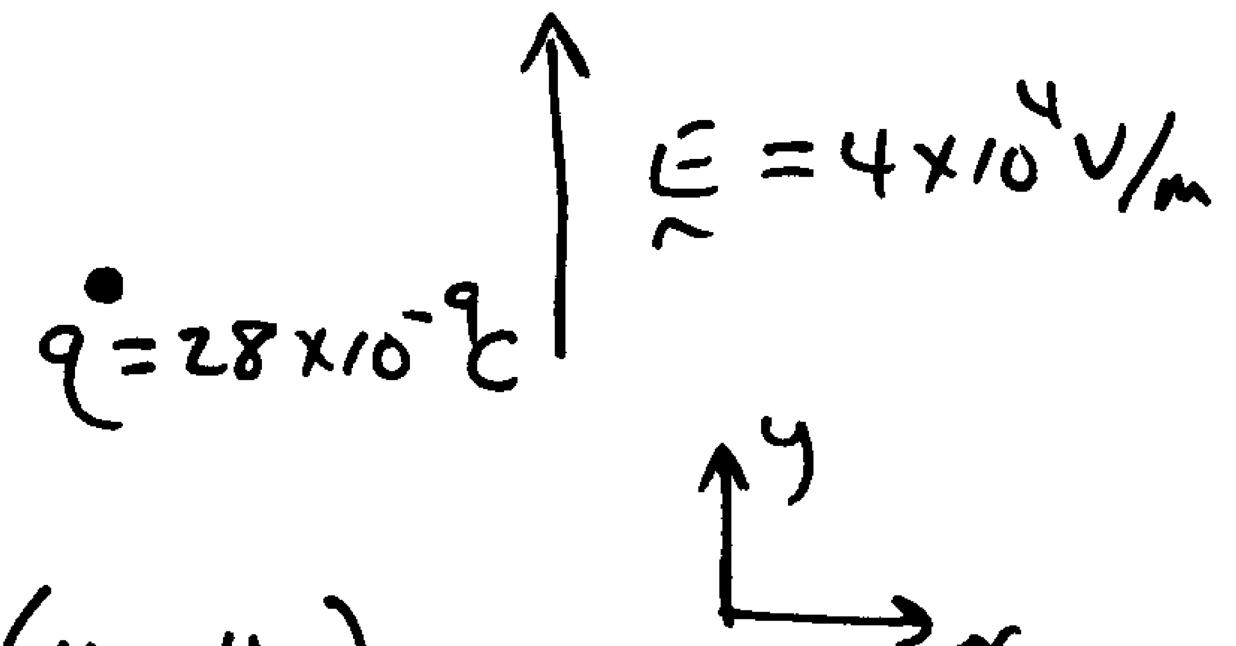
E&M Problem Sheet 3 Solutions

(1) $W_{\text{done by } \underset{\sim}{E}} = \int_a^b \underset{\sim}{F} \cdot d\underset{\sim}{l} = \int_a^b q\underset{\sim}{E} \cdot d\underset{\sim}{l}$



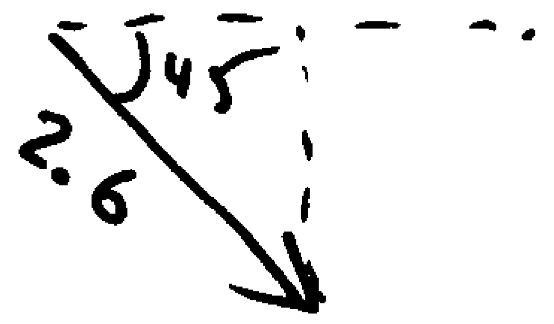
Here $\underset{\sim}{E}$ is uniform, so

$$W_{a\to b} = q\underset{\sim}{E} \cdot \underset{\sim}{l} = qE_y\hat{y} \cdot \underset{\sim}{l} = qE\times(y_b - y_a)$$

a) 0.450 m to right has $y_b = y_a$ $\therefore W_{a\to b} = 0$
(ie $\underset{\sim}{l}$ ⊥'r $\underset{\sim}{E}$)

b) 0.670 m upward $\Rightarrow W_{a\to b} = qE\times0.670 = 28\times10^{-9}\, 4\times10^{4}\, 0.670$
$= 7.50\times10^{-4}$ J

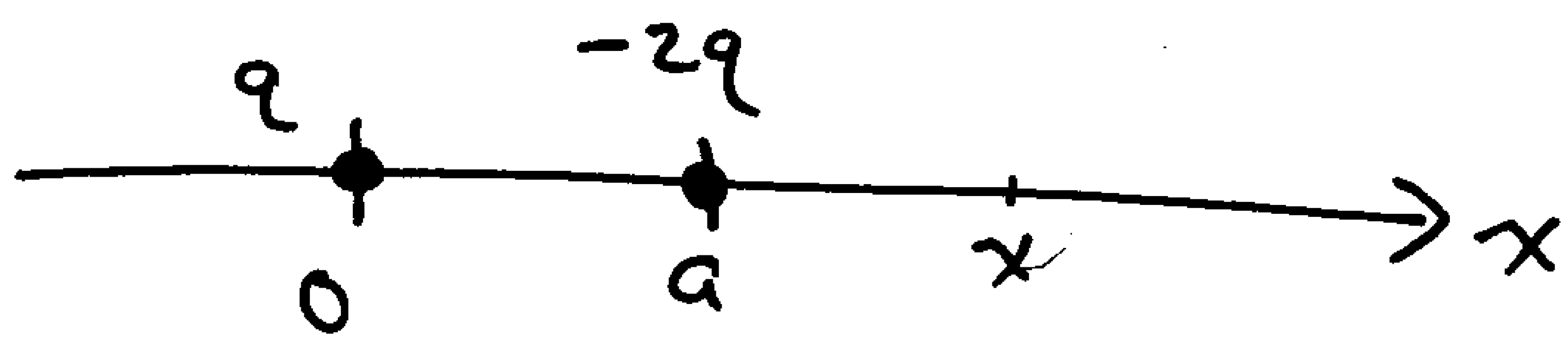
c) 2.6 m downward 45° from horizontal:



$y_b - y_a = -2.6\sin45° = -2.6\frac{\sqrt{2}}{2}$

so $W_{a\to b} = 28\times10^{-9}\, 4\times10^{4}\left(-2.6\frac{\sqrt{2}}{2}\right) = -2.06\times10^{-3}$ J

(2) a) as shown



b) By superposition

$$V(x) = V_q + V_{-2q} = \frac{q}{4\pi\epsilon_0}\frac{1}{|x-0|} + \frac{-2q}{4\pi\epsilon_0}\frac{1}{|x-a|}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|x|} - \frac{2}{|x-a|}\right]$$

Note as $|x|\to\infty$ $V\to0$ as required.

c) $V=0$ requires $\frac{1}{|x|} = \frac{2}{|x-a|}$ or $|x-a| = 2|x|$

i) $x>a \Rightarrow x-a = 2x \Rightarrow x=-a$ which is outside $x>a$ X

$0<x<a \Rightarrow -x+a = 2x \Rightarrow \underline{x = a/3}$

$x<0 \Rightarrow -x+a = -2x \Rightarrow \underline{\underline{x=-a}}$

[plus, of course, $x\to\pm\infty$]

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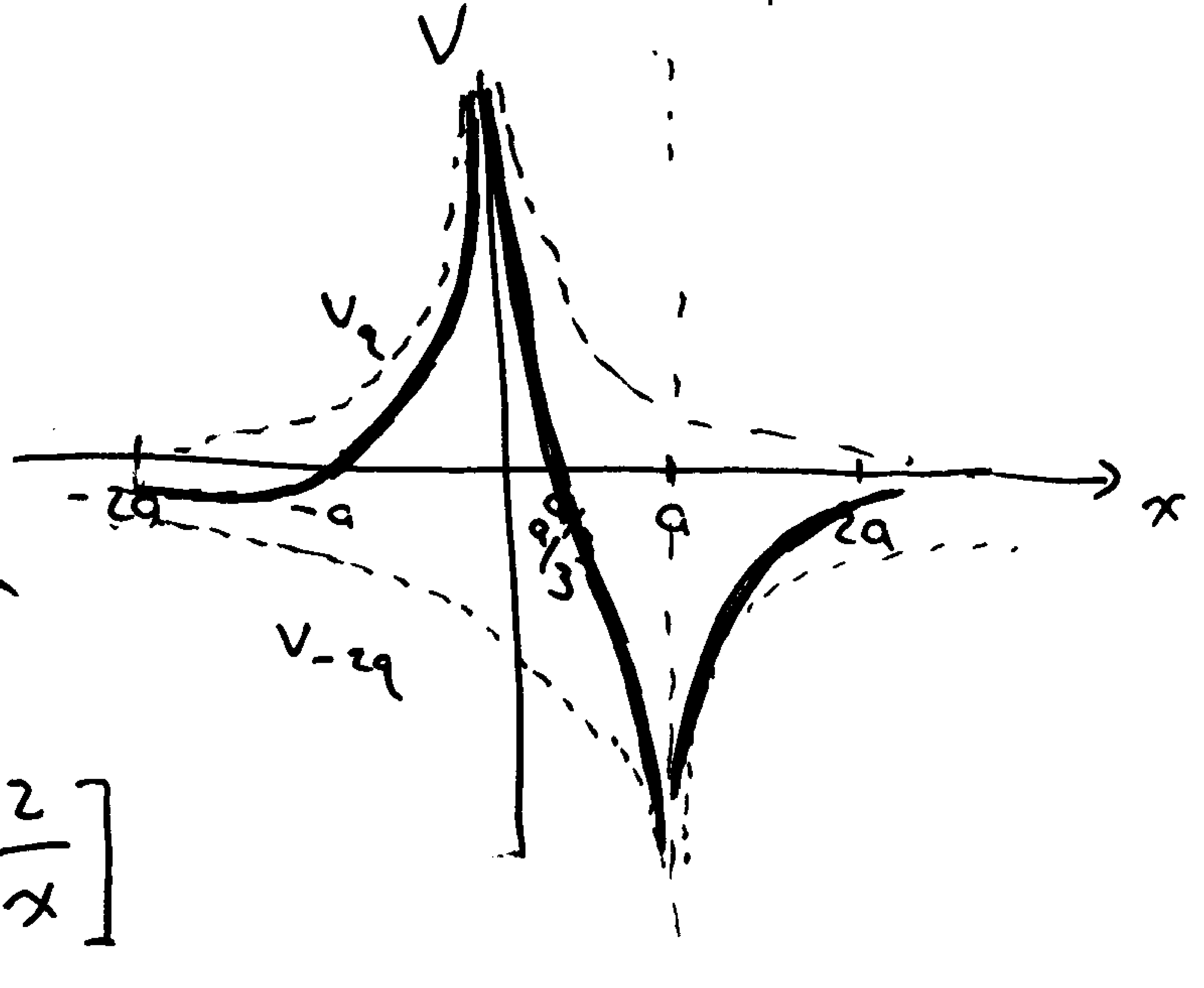
(2) d) Graph as shown.

Note asymptotes at $x=0$, $x=a$,



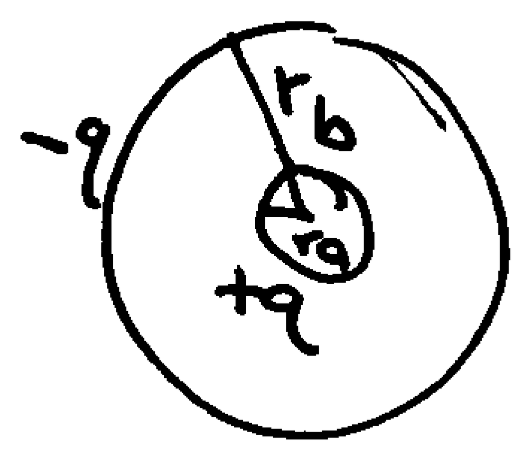
e) for $x \gg a$ $V(x)$ from (b) becomes

$$V(x) \approx \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x} - \frac{2}{x}\right]$$

$$= -\frac{q}{4\pi\epsilon_0}\frac{1}{x}$$

which is $V(x)$ for a pt charge $-q$ at origin. This is expected, as for $x \gg a$ configuration looks like a single charge $(+q)+(-2q) = -q$

(3) By symmetry $V = V(r)$



Outside a spherically symmetric charge distribution $\underset{\sim}{E}$ (by Gauss) and hence V are same as that with all the charge at centre. For inner metal (= conducting) sphere, charge will be on surface, $\underset{\sim}{E}_{inside} = \underset{\sim}{0} \Rightarrow V = \text{constant}$.

a) $V_{+q} = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$ $r > r_a$; $V_{-q} = \frac{-q}{4\pi\epsilon_0}\frac{1}{r}$ $r > r_b$

so iii) $r > r_b$ $V = V_{+q} + V_{-q} = \frac{q}{4\pi\epsilon_0}\frac{1}{r} - \frac{q}{4\pi\epsilon_0}\frac{1}{r} = 0$

ii) $r_a < r < r_b$ $V = V_{+q} + \text{const}$ where const ensures that $V(r)$ continuous at $r = r_b$

so $V = \frac{q}{4\pi\epsilon_0}\frac{1}{r} - \frac{q}{4\pi\epsilon_0}\frac{1}{r_b}$ so $V(r_b) = 0$

i) $r < r_a$ $V = \text{const} = V(r_a) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$

(3) b) $V_{ab} = V(r_a) - V(r_b) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) - 0$

$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$

c) For $r_a < r < r_b$ $\quad E_r = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{r_b}\right)$

ie $E_r = +\frac{q}{4\pi\epsilon_0}\frac{1}{r^2}$

From (b) $\frac{q}{4\pi\epsilon_0} = \frac{V_{ab}}{1/r_a - 1/r_b}$ so $\underline{E_r = \frac{V_{ab}}{1/r_a - 1/r_b}\frac{1}{r^2}}$

(4) $V = x^2 + 3y^2 - 2xz^2$

a) $\underset{\sim}{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$

$\underline{= -(2x - 2z^2)\hat{x} - 6y\hat{y} - 4xz\hat{z}}$

b) At $z=0$ $\quad V = x^2 + 3y^2 \quad \underset{\sim}{E} = -2x\hat{x} - 6y\hat{y}$

so equipotentials $V = \text{const}$ are ellipses: $1 = \frac{x^2}{V} + \frac{y^2}{V/3}$

$V=1: 1 = x^2 + \frac{y^2}{1/3} \quad V=3: 1 = \frac{x^2}{3} + y^2 \quad V=9: 1 = \frac{x^2}{9} + \frac{y^2}{3}$