First Year Electricity & Magnetism **Problem Sheet 3** **11 February 2008**

Electric Potential

Numbers in italics refer to the Exercise number in the 12th Edition of Young & Freedman's "University Physics". Answers to odd-numbered questions are at the back of the book.

1. [*23.17*] A charge of 28.0 nC is placed in a uniform electric field that is directed vertically upward and has a magnitude of 4.00×10^4 V/m. What work is done by the electric force when the charge moves (a) 0.450 m to the right; (b) 0.670 m upward; (c) 2.60 m at an angle of 45.0° downward from the horizontal?

2. [*23.25*] A positive charge q is fixed at the point $x = 0$, $y = 0$, and a negative charge $-2q$ is fixed at the point $x = a$, $y = 0$. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential V at points on the x-axis as a function of the coordinate x. Take V to be zero at an infinite distance from the charges. (c) At which positions on the x-axis is $V = 0$? (d) Graph V at points on the x-axis as a function of x in the range from $x = -2a$ to $x = +2a$. (e) What does the answer to part (b) become when $x \gg a$? Explain why this result is obtained.

3. [*23.49*] A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge $-q$ on the outer spherical shell. (a) Calculate the potential $V(r)$ for (i) $r < r_a$; (ii) $r_a < r < r_b$; (iii) $r > r_b$. (Hint: The net potential is the sum of the potentials due to the individual spheres.) Take V to be zero when r is infinite. (Hint: You may find it easier to do these in the order (iii), (ii), (i)) (b) Show that the potential of the inner sphere with respect to the outer is

$$V_{ab} = \frac{q}{4\pi\epsilon_o}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(c) Use $E_r = -\partial V/\partial r$ and the result from (a) to show that the electric field at any point between the spheres has magnitude

$$E(r) = \frac{V_{ab}}{(1/r_a - 1/r_b)}\frac{1}{r^2}$$

[You may attempt parts (d) and (e) of this question if you wish.]

4. In a certain region of space, the electric potential is $V(x, y, z) = x^2 + 3y^2 - 2xz^2$ Volts. (a) Calculate the x-, y- and z-components of the electric field. (b) In the plane $z = 0$ sketch the equipotentials corresponding to $V = 1$, $V = 3$, and $V = 9$ Volts together with the electric field lines.

Some possibly useful information and formulae are given overleaf

Some possibly useful information and formulae. You might find it useful to annotate these formulae with what they mean, where they come from, and what limitations or assumptions are involved - so you can tell whether they apply or not to any given situation.

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\begin{aligned}
W_{a\to b} &= \int_a^b \mathbf{F}\cdot\mathbf{d}\boldsymbol{\ell} \\
W_{a\to b} &= -(U_b - U_a) \\
U(r) &= \frac{qq_o}{4\pi\epsilon_o}\frac{1}{r} \\
U &= \frac{1}{4\pi\epsilon_o}\sum_{i<j}\frac{q_i q_j}{r_{ij}} \\
V &= \frac{U_{q_o}}{q_o} \\
V &= \frac{1}{4\pi\epsilon_o}\int\frac{dq}{r_q} \\
V_a - V_b &= +\int_a^b \mathbf{E}\cdot\mathbf{d}\boldsymbol{\ell} = -\int_b^a \mathbf{E}\cdot\mathbf{d}\boldsymbol{\ell} \\
\mathbf{E} &= -\nabla V
\end{aligned}
$$

Symbol	Value	Units
ϵ_0	8.85×10^{-12}	$C^2/N\ m^2$
$1/4\pi\epsilon_0$	8.99×10^{9}	$N\ m^2/C^2$
m_e	9.11×10^{-31}	kg
e	1.60×10^{-19}	C

Steve Schwartz

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