First Year Electricity & Magnetism **Problem Sheet 2** **5 Feb 2008**

Gauss's Law

Numbers in italics refer to the Exercise number in the 12th Edition of Young & Freedman's "University Physics". Answers to odd-numbered questions are at the back of the book.

1. [*22.31*] A negative charge $-Q$ is placed inside the cavity of a hollow metal solid. The outside of the solid is grounded, by connecting a conducting wire between it and the earth. a) Is there any excess charge induced on the inner surface of the piece of metal? If so, find its sign and magnitude. b) Is there any excess charge on the outside of the piece of metal? Why, or why not? c) Is there an electric field in the cavity? Explain. d) Is there an electric field within the metal? Why, or why not? Is there an electric field outside the piece of metal? Explain why or why not d) Would someone outside the solid measure an electric field due to the charge $-Q$? Is it reasonable to say that the grounded conductor has *shielded* the region from the effects of the charge $-Q$. In principle, could the same thing be done for gravity. Why or why not?

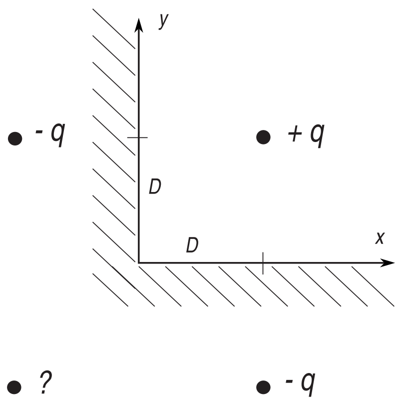
2. [*22.37*] A long coaxial cable consists of an inner cylindrical conductor with radius a and an outer coaxial cylinder with inner radius b and outer radius c. The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a uniform positive charge per unit length λ. Calculate the electric field a) at any point between the cylinders, a distance r from the axis; b) at any point outside the outer cylinder. c) Graph the magnitude of the electric field as a function of the distance r from the axis of the cable, from $r = 0$ to $r = 2c$. d) Find the charge per unit length on the inner surface and on the outer surface of the outer cylinder.

3. [*22.66 (a) and (b) only*] A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density $\rho(r)$ is given by

$$\begin{aligned} \rho(r) &= \alpha && \text{for} \quad r \le R/2 \\ \rho(r) &= 2\alpha(1 - r/R) && \text{for} \quad R/2 \le r \le R \\ \rho(r) &= 0 && \text{for} \quad r \ge R \end{aligned}$$

   Here α is a positive constant having units of $\mathrm{C/m^3}$. a) Determine α in terms of Q and R. b) Using Gauss's Law, derive an expression for the magnitude of $\mathbf{E}$ as a function of r. Do this separately for all three regions. Express your answers in terms of the total charge Q. Be sure to check that your results agree on the boundaries of the regions. [*Hint: The volume of a spherical shell of thickness dr at radial position r is $dV = 4\pi r^2 dr$. This is challenging but worth the effort.*]

4. A corner conductor is formed by grounded, conducting plates occupying the $y < 0$ portion of the $x - z$ plane and the $x < 0$ portion of the $y - z$ plane as shown in the accompanying diagram. A charge $+q$ is placed at the position $x = D$, $y = D$. [You may take $z = 0$ from here and treat the problem in the $x - y$ plane.] Considering first each plate separately suggests that image charges $-q$ should be located at $(x, y) = (-D, +D)$ and $(x, y) = (+D, -D)$. a) Show that these are not sufficient to satisfy the condition that the net electric field has no component tangential to the conducting surface. [*Hint: consider the region along the x-axis near the origin, for example*]. b) Show that the addition of an image charge at the point $(x, y) = (-D, -D)$ can resolve this problem and determine its charge. [*You may notice some similarity between the solution to this problem in terms of positive and negative (reversed) images and the optical result of standing in front of a corner mirror.*]



Some possibly useful information and formulae:

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

$$\Phi_E = \oint\!\!\oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{encl}}{\epsilon_0}$$

$$Q = \iiint \rho \, dV$$

Symbol	Value	Units
ϵ_0	8.85×10^{-12}	$\mathrm{C^2/N\,m^2}$
$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\,m^2/C^2}$

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