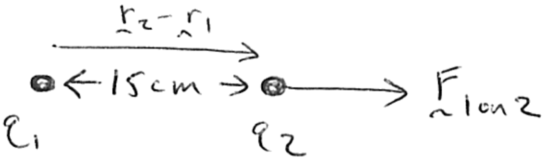
E & M PS1 Solutions 2008



1) From Coulomb's force law

$$\underset{\sim}{F}_{1on2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\underset{\sim}{r}_2-\underset{\sim}{r}_1|^2}\underbrace{\widehat{(\underset{\sim}{r}_2-\underset{\sim}{r}_1)}}_{\text{unit vector}}$$

so $$|\underset{\sim}{F}_{1on2}| = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\underset{\sim}{r}_2-\underset{\sim}{r}_1|^2}$$

Here $|\underset{\sim}{F}_{1on2}| = 0.220N$ and $|\underset{\sim}{r}_2-\underset{\sim}{r}_1| = 0.15m$, so

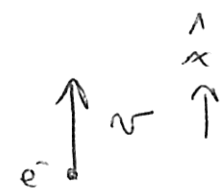
a) $q_1 = q_2 = q \Rightarrow \dfrac{0.220N}{8.99\times10^{+9}Nm^2/C^2}(0.15m)^2 = q^2 = 5.51\times10^{-13}C^2$

so $q = 7.42\times10^{-7}C = 0.742\mu C$

b) Now $q_1 = 4q_2 = 4q$. Repeating gives $4q^2 = 5.51\times10^{-13}C^2$

so $q = 3.71\times10^{-7}C$; $q_1 = 4q = 1.48\times10^{-6}C$

2) a) Newton $m_e\dfrac{d\underset{\sim}{v}}{dt} = \underset{\sim}{F} = q\underset{\sim}{E} = -e\underset{\sim}{E}$



Acceleration is vertically up, so there can only be a vertical component of $\underset{\sim}{E}$. Hence

$m_e\dfrac{dv}{dt} = -eE \Rightarrow$ ~~[illegible]~~ $v = -\dfrac{eE}{m_e}t + \underbrace{\cancel{v_0}}_{0 \text{ at rest at } t=0}$

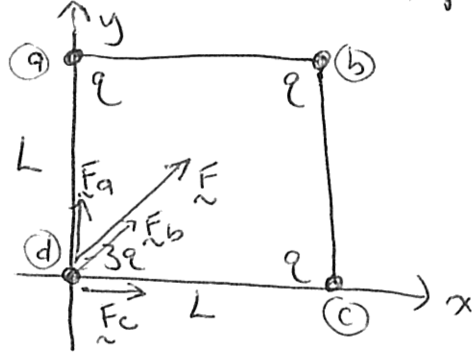
Hence $x = \int v\,dt = -\dfrac{eE}{m_e}\dfrac{t^2}{2} + \underbrace{\cancel{x_0}}_{\text{take}=0 \text{ w.l.o.g.}}$

so from numbers $4.5\text{ m} = \dfrac{-1.6\times10^{-19}C}{9.11\times10^{-31}kg}\dfrac{(3\times10^{-6}s)^2}{2}E$

$\Rightarrow E = -5.69\ V/m$ [− sign means $\underset{\sim}{E}$ is downward $\downarrow\underset{\sim}{E}$]

b) Compare $\left|\dfrac{m_e g}{eE}\right| = \dfrac{9.11\times10^{-31}\times9.8}{1.6\times10^{-19}\ 5.69} = 9.8\times10^{-12} \ll 1$

so grav. force on e^- <u>is</u> neglible.



3) Set up on x-y plane with $(-3q)$ at O

Then $\underset{\sim}{F}_{\text{on } -3q} = \underset{\sim}{F}_a + \underset{\sim}{F}_b + \underset{\sim}{F}_c$ by superposition

Now $\underset{\sim}{F}_{a \text{ on } d} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_a q_d}{L^2} \dfrac{(\underset{\sim}{r}_d - \underset{\sim}{r}_a)}{|\underset{\sim}{r}_d - \underset{\sim}{r}_a|}$

$= \dfrac{1}{4\pi\epsilon_0} \dfrac{q(-3q)}{L^2} \dfrac{\underset{\sim}{0} - L\hat{y}}{L} = \dfrac{3q^2}{4\pi\epsilon_0} \dfrac{1}{L^2} \hat{y}$

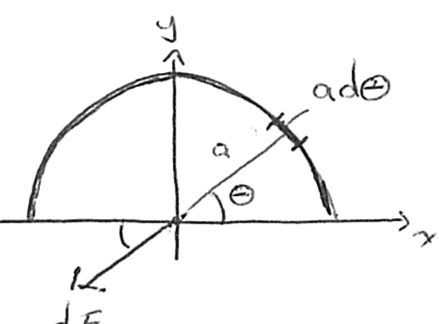
Similarly $\underset{\sim}{F}_c = \dfrac{3q^2}{4\pi\epsilon_0} \dfrac{1}{L^2} \hat{x}$ and $\underset{\sim}{F}_b = \dfrac{3q^2}{4\pi\epsilon_0} \dfrac{1}{2L^2} \left(\dfrac{\sqrt{2}}{2} \hat{x} + \dfrac{\sqrt{2}}{2} \hat{y} \right)$

So $\underset{\sim}{F}_{\text{on } -3q} = \dfrac{3q^2}{4\pi\epsilon_0} \dfrac{1}{L^2} \left(\hat{y} + \hat{x} + \dfrac{\sqrt{2}}{4} \hat{x} + \dfrac{\sqrt{2}}{4} \hat{y} \right) = \dfrac{3q^2}{4\pi\epsilon_0 L^2} \left(1 + \dfrac{\sqrt{2}}{4}\right)(\hat{x} + \hat{y})$

Hence $|\underset{\sim}{F}_{\text{on } -3q}| = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(1 + \dfrac{\sqrt{2}}{4}\right)\sqrt{2} = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right) = \underline{\dfrac{3q^2}{4\pi\epsilon_0} \dfrac{1}{2L^2} (1 + 2\sqrt{2})}$

Direction is $(\hat{x} + \hat{y})/\sqrt{2}$, ie toward centre of square

Vectors as shown.



4) Charge/length $\lambda = \dfrac{Q}{\pi a}$ so $dQ = \dfrac{Q}{\pi a} a\,d\theta$

At origin, dQ produces $d\underset{\sim}{E}$ as shown, namely $d\underset{\sim}{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{dQ}{a^2} (-\cos\theta\,\hat{x} - \sin\theta\,\hat{y})$

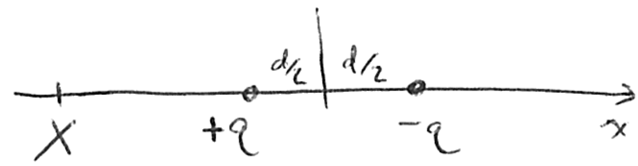
By symmetry, the sum of $dE_x \to 0$, so just need

$E_y = \sum dE_y = \sum \dfrac{1}{4\pi\epsilon_0 a^2} \dfrac{Q}{\pi} d\theta(-\sin\theta) = \dfrac{Q}{4\pi^2\epsilon_0 a^2} \int_0^{\pi} -\sin\theta\, d\theta$

$= \dfrac{Q}{4\pi^2\epsilon_0 a^2} \cos\theta \Big|_0^{\pi} = -\dfrac{Q}{2\pi^2\epsilon_0 a^2}$

So $|\underset{\sim}{E}| = +\dfrac{Q}{2\pi^2\epsilon_0 a^2}$ and direction $= -\hat{y}$

5) Given



$$\underset{\sim}{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{-1}{(x+\frac{d}{2})^2} + \frac{1}{(x-\frac{d}{2})^2}\right]\hat{x}$$

Now $(x \pm \frac{d}{2}) \equiv x(1 \pm \frac{d}{2x})$ so $(x \pm \frac{d}{2})^{-2} = x^{-2}(1 \pm \frac{d}{2x})^{-2}$

use binomial expansion : $(1 \pm \frac{d}{2x})^{-2} \approx 1 - 2(\pm\frac{d}{2x}) + \ldots$

Hence $$\underset{\sim}{E} \approx \frac{q}{4\pi\epsilon_0}\frac{1}{x^2}\left[-1(1-\frac{d}{x}) + 1(1+\frac{d}{x})\right]\hat{x}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{x^2}\left[0 + \frac{2d}{x}\right]\hat{x} = \frac{qd\,2}{4\pi\epsilon_0 x^3}\hat{x} = \frac{qd}{2\pi\epsilon_0 x^3}\hat{x}$$

Check Direction: Since $x<0$, $x^3<0$ so $\underset{\sim}{E}$ in $-\hat{x}$. That makes sense since $+q$ is closer, so it wins over $-q$.

Check dependence: A single charge would give $E \propto 1/x^2$. But here there is a nearby opposite charge so they cancel to leading order, leaving only a fraction $\sim|d/x|$ left